Intensity-dependent dispersion under conditions of electromagnetically induced transparency in coherently prepared multistate atoms

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Abstract
Interest in lossless nonlinearities has focussed on the dispersive properties of systems. From these coefficients we may easily determine the intensity dependent dispersion as a measure for these nonlinear optical properties. The nonlinearity of these systems increases as the number of atomic states increases, whilst the EIT transparency is maintained. It therefore appears likely that such multistate systems will be useful in the search for new quantum nonlinear optical materials.

1. Introduction
There is increasing interest in studying non-linear optical effects that turn on at the single or few photon level – Quantum Non-Linear Optics. Electromagnetically Induced Transparency (EIT) has been consistently discussed as a method of providing the appropriate conditions. Most of the focus of these investigations has been on atoms in the three level A configuration. Although dramatic demonstrations of non-linear effects have been shown (for example slow, stopped and superluminal light) (Matsko et al. 2001), these effects have been cross modulation experiments, where a strong field affects a weak field. It is difficult to achieve self or weak field weak field modulation and so attention has shifted towards multi-level atoms for such properties. The N configuration has been studied for a variety of purposes, and has been extended to a ‘chain N’ configuration for enhanced non-linearity (Zubairy et al. 2002).

Unfortunately such systems all suffer from the same dilemma, namely that resonance is required for large non-linearities, but simultaneously, this resonance increases dephasing because an excited state is always mixed into the superposition. Here we present an alternative, where the non-linearity is achieved in a system where only ground states contribute to the superposition. We term this the Chain A configuration and describe some of its properties and practical schemes for realization. This work has appeared in publication (Greentree et al. 2001)

2. System
The Chain A system is a multi-state atom. The lowest order chain A is the standard A configuration (Fig. 1(a)) with ground states |g⟩, |g⟩ and excited state |e⟩. The next Chain A is the M configuration, formed by adding states |e⟩ and |e⟩ as in Fig. 1(b). One can then generalize this to any arbitrary order of system. In all cases there are selection rules such that the probe excites transitions of the form |g⟩ → |e⟩ and the coupling field excites transitions of the form |g⟩ → |e⟩.

Fig. 1: Energy-level configurations for chain A atoms. (a) is the usual system, (b) is the 3-state chain A M system, (c) shows the generalization to a higher number of states.

This configuration is fine for atomic media, for example as alkali vapour in Doppler-limited the M system in an F = 2 to F = 2 transition. The coupling field is then polarized and turned on before the F = 2 polarized probe. Optical pumping then selects the desired M system over the undesired W system. This scheme generalizes in an obvious fashion. This method is not significantly more complex than the standard schemes used for the observation of EIT in A systems.

Fig. 2: A possible way to realize the M system using an F = 2 to F = 2 transition. The coupling field is then polarized and turned on before the F = 2 polarized probe. Optical pumping then selects the desired M system over the undesired W system.

3. Hamiltonian
The Hamiltonian for the Chain A is surprisingly easy to investigate and has a tridiagonal form. In the rotating wave approximation, we can write this:

\[ H = \sum_{j=1}^{N} \left( \frac{1}{2} \Delta_j \sigma_j^+ \sigma_j^- + \frac{1}{2} \Gamma_j \sigma_j^+ \sigma_j^+ + \frac{1}{2} \Gamma_j \sigma_j^- \sigma_j^- \right) \]

where \( \Delta_j \) is the probe (coupling) detuning, and \( \Gamma_j \) is the probe (coupling) Rabi frequency on the \( j \) th transition. We express this Hamiltonian in matrix form and proceed to investigate the eigenstates in the vicinity of resonance, i.e. \( \Delta_j = \Delta_j = \Delta_k \). This is more important to concentrate on the overall shapes of the graphs, rather than the absolute values. These clearly show a strong nonlinear effect, and increasing nonlinearity with increasing order of chain A.

4. Eigenspectra
In Figs. 3(a), (b) and (c) we show the eigenspectra associated with the 3-, 5- and 7-state chain A configurations respectively as a function of F with \( P = P \) and \( C = C \) for all \( l \). Note that in each Figure there is a state at zero energy, which corresponds to the ground state. By maintaining our system in this state, we prevent decoherence.

Fig. 3: Eigenspectra as a function of F for (a) 3-state, (b) 5-state and (c) 7-state chain A systems. Note the non-radiative progression of the invariant dressed states on (b) and (c), and the presence of the ground state with Energy 0.

5. Intensity Dependent Effects
To analytically investigate intensity dependent effects, we investigate the composition of the ground state. Unnormalized coefficients expressing the probability are presented in the table for 3-, 5- and 7-state chain A systems. From these coefficients we may easily determine the intensity dependent dispersion.

For a given material and configuration, it is possible to calculate an intensity dependent group velocity. Using standard values for Rubidium we have calculated values for the group velocity as a function of \( P \), although it is more important to concentrate on the overall shapes of the graphs, rather than the absolute values. These clearly show a strong nonlinear effect, and increasing nonlinearity with increasing order of chain A.

Fig. 4: Graphs showing dispersion (times linewidth squared) against \( P \) for a function of \( P \). It is more important to concentrate on the overall shapes of the graphs, rather than the absolute values. These clearly show a strong nonlinear effect, and increasing nonlinearity with increasing order of chain A.

6. Probe Susceptibility
We have also calculated probe susceptibility spectra (real and imaginary parts) using a density matrix approach for the 3-state and 5-state chain A systems. These are presented in Fig. 6 and should be compared with the eigenspectra presented in Fig. 3.

Fig. 5: Graphs of group velocity vs. \( P \) for different values of \( P \) for 3-, 5-, 7- and 9-state chain A systems.

We have also calculated probe susceptibility spectra (real and imaginary parts) using a density matrix approach for the 3-state and 5-state chain A systems. These are presented in Fig. 6 and should be compared with the eigenspectra presented in Fig. 3.

Fig. 6: Probe susceptibility (a) real and (b) imaginary for the 3-state chain A, (c) real and (d) imaginary for the 5-state chain A.

Conclusions
We have shown that there exist interesting nonlinear properties for chain A atoms, and in particular, we have focussed on the intensity-dependent dispersion as a measure for these nonlinear optical properties. The nonlinearity of these systems increases as the number of atomic states increases, whilst the EIT transparency is maintained. It therefore appears likely that such multistate systems will be useful in the search for new quantum nonlinear optical materials.

References