The use of a spectrally resolved interferometric correlation to analytically determine the phase of two unknown ultrashort laser pulses

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The simultaneous characterization of two unknown ultrashort laser pulses has been usually carried out using XFROG [1], and a retrieval algorithm [2]. In this paper we will describe a new technique that allows, for the first time, an analytical determination of the complex electric fields of two different pulses which have the same central frequency. This is done by performing a spectrally resolved analysis, in the Fourier domain, of a collinear cross-correlation. We call this technique "blind Measurement of Electric Field by Interferometric Spectral Trace Observation" (Blind-MEFISTO).

An example of a Blind-MEFISTO trace can be seen in figure 1 (a). This can be mathematically described as:

$$I^{SHG}(f,\tau) = \left| F_t\left\{ \left(E(t)\exp[i\,2\pi f_0 t] + G(t-\tau)\exp[i\,2\pi f_0 (t-\tau)]\right)^2 \right\}\right|^2 \qquad (1)$$

Where $E(t)$ and $G(t)$ are the slowly varying amplitude of the complex electric field centred at a frequency f_0. The Fourier transform with respect to the variable t is indicated by F_t. The collinear configuration allows to achieve phase matching for all the cross-terms in (1) and, as we will show, these terms will make possible to analytically obtain $E(t)$ and $G(t)$ simultaneously (blind-correlation). In order to do this, we first calculate the Fourier transform of equation (1) in the τ axis, i.e., $Y^{SHG}(f,\kappa)=F_\tau\{I^{SHG}(f,\tau)\}$. This expression results in 5 main spectral components (see Fig. 1(b)) that are at frequencies DC, $\pm f_0$ and $\pm 2f_0$. For blind-MEFISTO we will focus upon the f_0 term. This can be written as,

$$Y^{SHG}_{\kappa\approx f_0}(f,\kappa) = 2E_{SHG}(f)E^*(f+f_0-\kappa)G^*(\kappa-f_0) + 2G^*_{SHG}(f)G(f+f_0-\kappa)E(\kappa-f) \qquad (2)$$

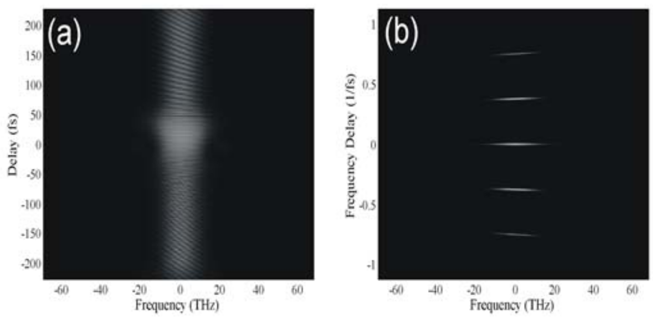

Fig. 1. a) Frequency resolved collinear cross-correlation of two unknown pulses. b) Same trace in the Fourier domain showing its Fourier components at frequencies DC, $\pm f_0$ and $\pm 2f_0$.

where $E_{SHG}(f)$ and $G_{SHG}(f)$ denote the second harmonic pulses. By writing all the involved complex magnitudes in polar form, i.e., $Y^{SHG}(f,\kappa)=R(f,\kappa)\exp[i\theta(f,\kappa)]$, $E(f)=U(f)\exp[i\varphi(f)]$ and $G(f)=V(f)\exp[i\gamma(f)]$, and after some algebra, one can see that using the information in two different slices at $\kappa = f_0$ and $\kappa = f_0-\Delta f$, it is possible to isolate the phase difference between consecutive phase components to obtain:

$$\Delta\varphi(f) = \pm\cos^{-1}\left[\Omega_1(f,\kappa=f_0)\right] \mp \cos^{-1}\left[\Omega_1(f,\kappa=f_0-\Delta f)\right] + \theta(f,\kappa=f_0) - \theta(f,\kappa=f_0-\Delta f) + \gamma(0) - \gamma(-\Delta f) \qquad (3)$$

$$\Delta\gamma(f) = \pm\cos^{-1}\left[\Omega_2(f,\kappa=f_0)\right] \mp \cos^{-1}\left[\Omega_2(f,\kappa=f_0-\Delta f)\right] - \theta(f,\kappa=f_0) + \theta(f,\kappa=f_0-\Delta f) + \phi(0) - \phi(-\Delta f) \qquad (4)$$

where we have defined,

$$\Omega_1(f,\kappa) = \frac{R^2(f,k) + 4U^2_{SHG}(f)U^2(f+f_0-\kappa)V^2(\kappa-f_0) - 4V^2_{SHG}(f)V^2(f+f_0-\kappa)U^2(\kappa-f_0)}{4R(f,\kappa)U_{SHG}(f)U(f+f_0-\kappa)V(\kappa-f_0)}.$$

and $\Omega_2(f,\kappa)$ is obtained interchanging $U(f)$ and $V(f)$. Note that all the variables included in $\Omega_1(f,\kappa)$ and $\Omega_2(f,\kappa)$ can be experimentally obtained. Equation (3) and (4) are the main result of this work.

In summary, this method allows for the first time, the analytical characterization of two pulses by determining the phase of $E(f)$ and $G(f)$. Blind-MEFISTO has the additional advantages in that it enables a simple extraction of pulse information without the need of an iterative retrieval algorithm and without having some of the ambiguities that are present in other techniques.

[1] K.W. DeLong, R. Trebino, and W.E. White, J. Opt. Soc. Am. B-Opt. Phys., 12 2463 (1995).
[2] D.J. Kane, G. Rodriguez, A.J. Taylor, and T.S. Clement, J. Opt. Soc. Am. B-Opt. Phys., 14 935 (1997).