Towards Spin Squeezing in Cold Atomic Ensembles

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For my mother, Anna
and in memory of my father, Stanisław
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Chapter 1

Introduction

The interaction of light with matter is of fundamental importance from scientific as well as technological point of view. Perhaps the story starts when our ancestors saw their own reflection for the first time and started using polished objects as mirrors. This is the story of optics, which describes how matter affects light. But it was probably not much later that humans realized that light focused by a piece of transparent material of a curved surface can be used to focus sunlight and ignite fire. In this case the energy of light was exploited to affect the state of the matter.

The construction of optical instruments allowed scientists to learn about the matter around us, from single biological cells to remote galaxies. With the advent of spectroscopy light became our fundamental tool for studying low-energy physics. The resolution of spectroscopic measurements was greatly improved with the invention of the laser. The extremely high intensity concentrated in a very narrow frequency band opened up a whole plethora of new possibilities, for both science and technology.

The unprecedented control over the properties of laser light encouraged scientists to think that experimental study of the interaction between single atoms and individual photons should be possible. The ubiquitous broadening of the atomic spectra due to Doppler effect was ultimately beaten when new methods of atom cooling became available. Instead of pushing cryogenic techniques, scientists used …light. Carefully controlled frequency and polarization of laser light facilitated trapping and cooling of atoms down to $\sim \mu$K- and even lower temperatures.
These great achievements moved us closer in the study of the quantum nature of light-atom interaction. However, despite the cross-section of such interactions being enormous ($\sim \lambda^2$) when compared to the size of an atom, it remains extremely difficult to observe the interaction of a single atom with a single photon. Meanwhile, the field of quantum information developed and this shifted somewhat the interest, from basic to applied.

A commonly implemented trick that enhances the strength of light-atom interaction consists in placing atoms in a cavity. Light bounces off the mirrors many times which increases the probability of interaction. The field of cavity-quantum electrodynamics is now well established and has proved a very good test bed for experimental tests of basic quantum mechanics. On the other hand, cavity-QED and other approaches are technically demanding or suffer from scalability and other problems. For this reason the quest for a physical system to implement quantum-information protocols that require an effective light-matter interface remains open. One such approach uses an ensemble of $\sim 10^{12}$ atoms if performed in a vapor cell at room temperature or $\sim 10^6$ atoms if they are cold and trapped. The numbers of photons are of similar order. The large number of particles used in these experiments does not make them ideal systems to study the interaction between single atoms and individual photons. Nevertheless, these mesoscopic systems can be used to manipulate quantum information in the regime of continuous variables.

By implementing this approach in a vapor cell setup, it has so far been possible to demonstrate some operations of fundamental interest in quantum information. These include entanglement of two distant, macroscopic objects [1], quantum memory for light [2] and teleportation between light and matter [3]. All these results have been produced by exploiting the interaction between a large collection of atoms prepared in a specific internal state and a detuned ($\Delta \sim 100 \Gamma - 1000 \Gamma$), multi-photon probe beam. The basic theoretical model showing the usefulness of this kind of system had been introduced earlier by Kuzmich et al [4]. In their paper they showed that in this regime the interaction is of QND type (quantum non-demolition) and proposed to use this QND Hamiltonian to prepare a squeezed state of atomic spin. At that time the potential advantages of using spin-squeezed states for spectroscopy tasks were known [5]. The concept of a spin-squeezed state had been introduced earlier by Wineland et al [6] and by
Kitagawa and Ueda [7].

Reduction of noise below the standard quantum limit or, more specifically, reduction of the quantum uncertainty of one of the two components perpendicular to the main polarization at the expense of an increase in the uncertainty of the other component could be used to improve not only the precision of atomic clocks [5] but also that of magnetometers [8]. The QND measurement can also be performed in a continuous fashion [9] and combined with real-time feedback [10]. Since the nature of the interaction is dispersive, the signal to be detected is the optical phase. This can be done in a Mach-Zehnder interferometer setup as in [11, 12] but most experiments exploit the phase difference acquired by the two circular components of a linearly polarized probe, which results in a rotation of the plane of polarization. This in turn can be detected in a simple polarimetric setup with balanced photodetection. Yet another detection method has been recently implemented by Schleier et al [13]. In this case the interaction is aided by a cavity and the transmission of the probe through the cavity is dependent upon the atomic spin.

The subject of this thesis has been the construction and characterization of a setup for studying the interaction of a detuned probe and an ensemble of cold atoms held in an optical dipole trap. Using cold atoms has several advantages. Most importantly interaction strength of comparable magnitude can be achieved with far fewer numbers of atoms [14] and with longer coherence times. To develop a feeling for the scaling consider the case of an ensemble of \( N_{\text{at}} = 10^6 \) atoms (dipole trap) and that of \( N_{\text{at}} = 10^{12} \) atoms (vapor cell). We assume that the two ensembles are completely polarized by which we mean that the components of the atomic spin perpendicular to the mean polarization are only due to the quantum uncertainty. We call \textit{classical} any signal that is proportional to the mean polarization and \textit{quantum} any signal that is proportional to the perpendicular components. It then follows that a classical signal scales proportionally with \( N_{\text{at}} \) while quantum signals scale proportionally with \( \sqrt{N_{\text{at}}} \). In our case all of these signals are carried by the probe beam and have to be measured by the detection system. We see that the reduction of the system size by a factor \( 10^6 \) results in a reduction of classical signals by the same factor while quantum signals are only reduced by a square of that, \( \sqrt{10^6} = 10^3 \). At first it looks as if reducing the size of the sample was a madman idea, even if one is concerned with only the quan-
tum signals, the detectors will have to be a 1000 times more sensitive! However, this discussion ignores the ever-present classical fluctuations which scale exactly in the same way as the classical signals and in fact can be regarded as a form thereof. Therefore in practice it may turn out that the ratio of the quantum signals to the classical signals, in this context called a signal to noise ratio (SNR), is more important than the magnitude of the (quantum) signal alone. Neglecting other sources of noise, the SNR for a system of $N_{\text{at}} = 10^{12}$ atoms will be $\sim 10^{-6}$, while the SNR of $N_{\text{at}} = 10^{6}$ atoms will be $N_{\text{at}} = 10^{-3}$. Thus a DC detection may be possible with a small sample while cell experiments typically require modulating the signal at a particular frequency and use lock-in detection techniques [1] which, of course, makes the experiment slower.

The remaining aspect of using cold atomic samples and a dipole trap is related to the geometry of the problem. As shall be shown in Ch. 2, the interaction strength $\kappa$ is inversely proportional to the area where the interaction takes place. Hence, a long and thin sample is better than short and fat one. In a red-detuned dipole trap the elongated geometry of the sample can be conveniently obtained by adjusting the size of the beam in its focal plane. Obviously, one could manufacture a long and thin vapor cell but in this case the collisions with the walls would be so frequent that the spin coherence time would be very much reduced. It is also possible to imagine a small probe beam interacting with a large vapor cell. However, in this situation in order to interact with all the atoms in the cell the probe pulse would have to be sufficiently long until all of them get a chance to cross the probe beam. Thus, again, the use of cold atoms in a dipole trap facilitates quick measurements. Typical numbers are $\sim 1$ ms for cell setups and $\sim 1 \mu s$ in dipole traps.

The thesis has the following layout. Chapter 2 starts with the introduction of the collective variables for atomic ensembles. In our case, these are the components a quasi-spin vector, that obey the usual commutation relations of angular momentum operators. Coherent and squeezed states are defined. The polarization of the probe is described in terms of the components of the Stokes vector, for which the commutator is of the same type. These definitions are used in the description of the light-atom interaction that follows. This starts with a simplified system and culminates with a presentation of the interaction in the particular case of $^{87}\text{Rb}$. The key experimental parameters are identified and the role of loss and extra
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noise is discussed. The chapter concludes with a discussion of two experimental methods that allow us to estimate the available interaction strength.

Chapter 3 starts with a brief discussion of the main cooling and trapping mechanisms. The construction of the MOT apparatus is only outlined as it has been presented elsewhere [15, 16]. On the other hand, the dipole trap is described in detail. The trap depth and frequencies are calculated. A simple estimate of the number of atoms that can be trapped is given. The loading conditions are discussed. The last sections show the results of lifetime and temperature measurements.

Chapter 4 describes the way the initial states of the ensemble and of the probe are prepared. The configuration of the beams is shown and their preparation discussed. A simplified argument to justify the size of the probe beam that matches the shape of the sample is given. The chapter concludes with a discussion of the experimental problem of alignment of the probe to the atomic sample.

Chapter 5 examines the problem of balanced shot-noise limited detection. It starts with a brief theoretical analysis which shows that balanced detection can be treated as a kind of homodyne measurement. Next the principal noise parameters of the photodiode like e.g. dark current are defined, and some basic properties of the detector as a whole e.g. the noise equivalent power are introduced. Later, basic designs of photodetector circuits are outlined and the concept of a charge sensitive amplifier as a solution to the problem of pulsed, low-noise photodetection is discussed. The photodetector circuit that has been constructed is analyzed in detail and its parameters are calculated and measured with emphasis on the noise characteristics. Finally, the range of shot-noise limited operation is carefully measured and possible application techniques and their influence on this range is discussed.

In chapter 6 we describe what has been a small project on its own. It consisted in constructing and testing a control unit that actively nulls stray magnetic fields with special emphasis on those at 50 Hz. This particular feature has been tested experimentally with cold atoms in the MOT as a sensor. The device has been designed in such a way that it also can be used to apply magnetic fields of arbitrary form. The measurement technique developed during the tests has been also applied as a diagnostic tool in a brief study of the dynamics of a magneto-optical trap.
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Chapter 7 reports first attempts to actively control and monitor the internal state of atoms trapped in our dipole trap. The MOT cooling and repumping light have been used to transfer the population between the two ground hyperfine levels. In order to verify the pumping procedure, an absorption spectroscopy setup has been built around the trapped sample. To avoid blowing the atoms out of the trap very low probe power has been used together with a reference beam that helped cancel intensity fluctuations. A very simple photodetector has been used in these experiments, but its gain and bandwidth had to be carefully tailored.

The main results of this thesis are presented in chapter 8. Using the optical pumping scheme described in Ch. 4 and the shot-noise limited balanced detection setup presented in Ch. 5 polarization rotation signals have been measured and the available interaction strength has been assessed from the measured rotation angle. In order to estimate the efficiency of the optical pumping, another polarization measurement, following the method explained in Sec. 2.4.7, has been performed. The results obtained using the two approaches are compared.

The concluding chapter 9 summarizes the progress of the experiment and suggests possible improvements in the experimental setup.
Chapter 2

Theoretical background

This chapter presents a basic theoretical description of the interaction between a collection of atoms in their ground state and an off-resonant, polarized probe. We begin by introducing the concepts of collective spin variables for atoms and that of the Stokes vector to characterize the polarization state of light. In this way the mathematical description of both atomic and light systems become very similar which proves useful at a later stage and facilitates the treatment of both systems on equal footing.

Having prepared the necessary notation we summarize the original proposal for spin squeezing by quantum non-demolition (QND) measurement in the X system [4]. This serves as an introduction and an illustration for the more complicated case of $^{87}$Rb which is discussed in the remainder of the chapter.

After a brief description of the $^{87}$Rb atom, we proceed to introduce the light-atom interaction Hamiltonian. We then bring our attention to the special but very important case of a large detuning for which the Hamiltonian turns out to be of the same form as that of the X system.

Following the presentation of the atom-light interaction, we introduce a technique that allows for a calibration of the interaction strength experimentally. As a final point we briefly discuss other applications of the interaction Hamiltonian in its more general form, in particular an atom number measurement.

The brief theoretical introduction presented here is far from exhaustive. For a detailed information on particular subjects the reader is referred to the bibliography given in the text. A detailed review that discusses atom-light interfaces from
a broader perspective can be found in Ref. [17].

## 2.1 Atomic state

The operator of the total angular momentum is usually denoted by $\hat{\mathbf{j}}$. $\hat{j}^2$ commutes with a component of $\hat{\mathbf{j}}$ in any direction, in particular with $\hat{j}_z$. The spectrum of $\hat{j}_z$ is discrete with eigenvalues $^1 m\hbar, m \in \{-j, -j+1, \ldots, j-1, j\}$ and the eigenvalue of $\hat{j}^2$ is $\hbar^2 j(j+1)$

$$\begin{align*}
\hat{j}_z |j, m\rangle &= m\hbar |j, m\rangle \\
\hat{j}^2 |j, m\rangle &= \hbar^2 j(j+1) |j, m\rangle.
\end{align*}$$

(2.1.1)

The vectors $|j, m\rangle$ form a complete orthogonal basis.

Any two of the components of the angular momentum vector $\hat{j}$ obey the commutator

$$[\hat{j}_r, \hat{j}_s] = i\hbar \epsilon_{rst} \hat{j}_t,$$

(2.1.2)

where $\{r, s, t\} \in \{x, y, z\}$ $^2$. For any two Hermitian operators $\hat{A}$ and $\hat{B}$ with a commutator $[\hat{A}, \hat{B}] = i\hat{C}$, where $\hat{C}$ is also Hermitian, we have

$$\text{var}(\hat{A})\text{var}(\hat{B}) \geq \frac{1}{4} \langle \hat{C} \rangle^2.$$  

(2.1.3)

Applying this relation to the components of the vector $\hat{j}$ results in an uncertainty relation that is of fundamental importance to this thesis

$$\text{var}(\hat{j}_r)\text{var}(\hat{j}_s) \geq \frac{\hbar^2}{4} \langle \hat{j}_t \rangle^2.$$  

(2.1.4)

From Eqs. 2.1.1 we have that the maximum value of $|m|$ is $j$ and the length of the vector $\hat{j}$ is $\sqrt{j(j+1)} \hbar$. It follows that, unless $j = 0$, $\hat{j}$ never points exactly along the $z$-axis. It is nearest to this situation if $m = \pm j$. Inequality 2.1.4 defines the best precision to which any two of the components of $\hat{j}$ can be measured simultaneously.

It is convenient to define the raising and lowering operators, $\hat{j}_+$ and $\hat{j}_-$

$$\hat{j}_\pm \equiv \hat{j}_x \pm i\hat{j}_y$$

(2.1.5)

$^1$See any quantum mechanics textbook. There also exist dedicated monographs that deal with the subject of angular momentum in quantum mechanics, e.g. [18], [19] or [20].

$^2$The usual notation for components of a vector in Cartesian coordinates is $\{i, j, k\}$. We choose a different set because $k$ is used later as an index enumerating atoms.
with the property
\[ \hat{J}_\pm |j, m\rangle = \hbar \sqrt{(j \mp m)(j \pm m + 1)} |j, m \pm 1\rangle \]  
\hspace{2cm} (2.1.6)

Applied to an eigenstate \( |j, m\rangle \) they raise/lower the magnetic quantum number \( m \) in unit steps within the \( j \)-manifold.

The operators defined so far are of general form. For instance \( \hat{J} \) may stand for the spin operator \( \hat{s} \), the orbital angular momentum operator \( \hat{l} \), the total angular momentum operator \( \hat{f} \) or any other angular momentum operator. As a convention small letters denote single particle operators.

We now introduce the notion of collective angular momentum operators. For an ensemble of \( N_{\text{at}} \) identical particles (in our case atoms) a collective angular momentum operator is denoted by the capital letter \( \hat{J} \) and defined by
\[ \hat{J} = \sum_{k=1}^{N_{\text{at}}} \hat{J}_k, \]  
\hspace{2cm} (2.1.7)

where \( k \) numerates the particles. As for the single particle operators we can write
\[ \hat{J}_z |J, M\rangle = M \hbar |J, M\rangle \]
\[ \hat{J}_z^2 |J, M\rangle = \hbar^2 J(J + 1) |J, M\rangle \]  
\hspace{2cm} (2.1.8)

with \( M \in \{-J, -J + 1, \ldots, J - 1, J\} \).

Collective operators obey the same commutation and uncertainty relations as single particle operators
\[ [\hat{J}_r, \hat{J}_s] = i\epsilon_{rst} \hat{J}_t, \]
\[ \text{var}(\hat{J}_r)\text{var}(\hat{J}_s) \geq \frac{1}{4}(\hat{J}_r)^2. \]  
\hspace{2cm} (2.1.9)

Likewise, collective raising and lowering operators, \( \hat{J}_+ \) and \( \hat{J}_- \), can be defined
\[ \hat{J}_\pm \equiv \hat{J}_x \pm i\hat{J}_y, \]  
\hspace{2cm} (2.1.10)

with
\[ \hat{J}_\pm |J, M\rangle = \hbar \sqrt{(J \mp J)(J \pm M + 1)} |J, M \pm 1\rangle. \]  
\hspace{2cm} (2.1.11)

2.1.1 Coherent spin states

Using 2.1.11, the variance of \( \hat{J}_x \) and \( \hat{J}_y \) can be computed in any eigenstate of \( \hat{J}_z \)
\[ \text{var}(\hat{J}_x) = \text{var}(\hat{J}_y) = \frac{\hbar^2}{2} [J(J + 1) - M^2]. \]  
\hspace{2cm} (2.1.12)
Clearly, the states $|J, \pm J\rangle$ have the smallest uncertainty,

$$\text{var}(\hat{J}_x)_{\text{CSS}} = \text{var}(\hat{J}_y)_{\text{CSS}} = \frac{\hbar^2}{2}. \quad (2.1.13)$$

Comparing this to the inequality 2.1.9 we see that these states are minimum uncertainty states. They are known as coherent spin states\(^3\) [7].

In general, for a large number of particles $N_{\text{at}}$ the decomposition of a collective state $|J, M\rangle$ in terms of the single particle states $|j, m\rangle$ is not trivial. Coherent states are an exception. In fact they are just product states

$$|J, \pm J\rangle = \bigotimes_{k=1}^{N_{\text{at}}} |j, \pm j\rangle_k, \quad (2.1.14)$$

Because in a product state all of the individual spins are independent, the variance of a coherent state is just a sum of all the individual variances

$$\text{var}(\hat{J}_x) = \sum_{k=1}^{N_{\text{at}}} \text{var}(\hat{j}_x^{(k)}) = N_{\text{at}} \frac{\hbar^2 j}{2} = \frac{\hbar^2 J}{2} \quad (2.1.15)$$

which confirms the earlier result.

### 2.1.2 Spin-squeezed states

In the general case we can remove the assumption that the only component of $\hat{J}$ that has a non-zero mean value is $\hat{J}_z$ and allow $\hat{J}$ to point in an arbitrary direction in space. A coherent state can then be represented as in Fig. 2.1(a). The length of the mean\(^4\) vector is $|\langle \hat{J} \rangle| = \hbar J$. The uncertainties in the normal plane form a circular disk with a radius $\sqrt{\frac{\hbar^2 J^2}{2}}$

The uncertainty disk can be deformed so that it becomes elliptic. If the semiminor axis of the ellipse is shorter than the radius of the uncertainty disk of a coherent state with the same $|\langle \hat{J} \rangle|$, the state is referred to as a spin-squeezed state. Unlike for a coherent state, in the case of a spin-squeezed state there exist quantum correlations among the individual spins\(^5\) [7].

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\(^3\)In this context the word spin is used for angular momentum of any kind.

\(^4\)It is emphasized that this is the mean vector. The length of the vector $\hat{J}$ is, of course, $\hbar \sqrt{J(J+1)}$.

\(^5\)It is also possible to introduce correlations between various forms of angular momentum within the same atom, [21]
The amount of squeezing is characterized by the squeezing parameter $\xi^2$ [5]

$$\xi^2 = \frac{\text{var}(\hat{J}_\perp)}{\text{var}(\hat{f}^{(\text{CSS})}_\perp) \langle \hat{f} \rangle^2} = 2 J \frac{\text{var}(\hat{f}_\perp)}{|\langle \hat{f} \rangle|^2}. \quad (2.1.16)$$

The fraction in the square ensures that the length $|\langle \hat{f} \rangle|$ is appropriately taken into account. A simple argument that justifies the square itself is as follows. It should be possible to define a dimensionless parameter that will allow for a direct comparison of the squeezed and coherent states. A natural choice is $\frac{\text{var}(\hat{f}_\perp)}{|\langle \hat{f}^{(\text{CSS})}_\perp \rangle|^2}$. Taking the ratio of this quantity for a squeezed and coherent state we obtain

$$\frac{\text{var}(\hat{f}_\perp)}{|\langle \hat{f} \rangle|^2} \frac{\text{var}(\hat{f}^{(\text{CSS})}_\perp)}{|\langle \hat{f}^{(\text{CSS})}_\perp \rangle|^2} = \xi^2. \quad (2.1.17)$$

The fraction $\frac{\text{var}(\hat{f}_\perp)}{|\langle \hat{f} \rangle|^2}$ is nothing but the square of the angle $\theta$ in Fig. 2.1. As such it characterizes the uncertainty of the direction in space in which $\hat{f}$ is pointing.

### 2.1.3 Pseudo-spin 1/2

The formalism developed so far is valid for “real” angular momentum operators or “spins” whose unit is $\hbar$. It is well known that a two-level system is equivalent to a pseudo-spin 1/2 [22]. We use this pseudo-spin notation to describe the relevant atomic states in the remainder of this chapter. The corresponding collective operators then characterize the entire atomic ensemble.
For convenience, we assume that the pseudo-spin is polarized along the $x$-axis and we are interested in minimizing the uncertainty of $\hat{J}_z$. Taking this into account, putting $j = 1/2$ and removing $h$, we can use the formalism developed so far to obtain the following expressions.

**Variance in a coherent pseudo-spin state**

$$\text{var}(\hat{J}_y) = \text{var}(\hat{J}_z) = \frac{1}{2}|\langle \hat{J}_x \rangle| = J/2 = N_{\text{at}}/4. \quad (2.1.18)$$

**Squeezing parameter (pseudo-spin)**

$$\xi^2 = \frac{\text{var}(\hat{J}_z)}{\langle \hat{J}_x \rangle^2} = \frac{4 \text{var}(\hat{J}_z)}{N_{\text{at}}^2}. \quad (2.1.19)$$

### 2.2 Polarization state of light

Having completed the description of the collective spin operators we turn our attention to the light field of the probe beam. The electric field of a single mode with two orthogonal polarizations can be expressed by

$$\hat{E} = \hat{E}^{(+)} + \hat{E}^{(-)} \quad (2.2.1)$$

where the positive and negative frequency parts are respectively

$$\hat{E}^{(+)} = \sqrt{\hbar g}(\hat{a}_+ e_+ + \hat{a}_- e_-) \exp[i(kz - \omega t)] \quad (2.2.2)$$

and

$$\hat{E}^{(-)} = (\hat{E}^{(+)})^\dagger. \quad (2.2.3)$$

$a_+$ and $a_-$ are the annihilation operators of the modes $'+$ and $'-$, respectively, $e_+$ and $e_-$ are unit polarization vectors defined later and $g \equiv \omega/(2\varepsilon_0 V)$ with $V$ being the mode volume. The annihilation operator and its conjugate creation operator obey the following commutation relation

$$[\hat{a}_q, \hat{a}_{q'}^\dagger] = \delta_{q,q'}. \quad (2.2.4)$$

where $\{q, q'\} \in \{+,-\}$.

This 1-dimensional description is correct if $A \gg \lambda^2$ with $A$ being the cross section of the beam. In what follows we omit the spatial exponential factor and

---

6see any of the many textbooks on quantum optics, e.g. [23]
write all expressions only at the point $z = 0$. This is justified by the fact that the probe beam is off-resonant and the atomic ensemble is relatively short so that the mutual effect of the probe and the atoms can be described by using collective variables as if the entire interaction happened at one point in space, that is the description becomes effectively 0-dimensional. The explicit time dependence is removed by going to rotating frame. Equation 2.2.2 can then be written as

$$
\hat{E}^{(+)} = \sqrt{\hbar g}(\hat{a}_+ e_+ + \hat{a}_- e_-)
$$

$$
\hat{E}^{(-)} = (\hat{E}^{(+)})^\dagger
$$

(2.2.5)

The symbols $\{+, -\}$ denote the circular polarization basis. The complete set of basis vectors is given by

$$
e_+ = \frac{1}{\sqrt{2}}(-e_x - ie_y)
$$

$$
e_- = \frac{1}{\sqrt{2}}(e_x - ie_y)
$$

$$
e_0 = e_z
$$

(2.2.6)

with the complex conjugate

$$
e_q^* = (-1)^q e_{-q}
$$

(2.2.7)

and the scalar product

$$
e_q^* \cdot e_{q'} = (-1)^q e_q \cdot e_{-q'} = \delta_{qq'}.
$$

(2.2.8)

### 2.2.1 Stokes vector

As will soon become clear, the quantity of interest is the polarization of light. Fully polarized light can be described using the Jones matrix formalism. However, the most general description is in terms of the Stokes vector (see e.g. [24]). Unlike Jones matrices, the Stokes vector formalism allows partial polarization.
The components of the Stokes vector are given by

\[ \hat{S}_0 = \frac{1}{2} (\hat{N}_{\text{ph},x} + \hat{N}_{\text{ph},y}) \]

\[ = \frac{i}{2} (\hat{a}_x^+ \hat{a}_x + \hat{a}_y^+ \hat{a}_y) \]

\[ = \frac{1}{2} (\hat{a}_+^+ \hat{a}_+ - \hat{a}_-^+ \hat{a}_-) \],

\[ \hat{S}_x = \frac{1}{2} (\hat{N}_{\text{ph},y} - \hat{N}_{\text{ph},x}) \]

\[ = \frac{1}{2} (\hat{a}_y^+ \hat{a}_y - \hat{a}_x^+ \hat{a}_x) \]

\[ = \frac{1}{2} (\hat{a}_+^+ \hat{a}_+ - \hat{a}_-^+ \hat{a}_-) \],

\[ \hat{S}_y = \frac{1}{2} (\hat{N}_{\text{ph},135^\circ} - \hat{N}_{\text{ph},45^\circ}) \]

\[ = \frac{1}{2} (-\hat{a}_y^+ \hat{a}_x - \hat{a}_x^+ \hat{a}_y) \]

\[ = \frac{i}{2} (\hat{a}_+^+ \hat{a}_+ - \hat{a}_-^+ \hat{a}_-) \],

\[ \hat{S}_z = \frac{1}{2} (\hat{N}_{\text{ph},\sigma^+} - \hat{N}_{\text{ph},\sigma^-}) \]

\[ = \frac{i}{2} (\hat{a}_y^+ \hat{a}_x - \hat{a}_x^+ \hat{a}_y) \]

\[ = \frac{1}{2} (\hat{a}_+^+ \hat{a}_+ - \hat{a}_-^+ \hat{a}_-) \].

(2.2.9)

Thus \( \hat{S}_0 \) represents the total number of photons, \( \hat{S}_x \) is the difference between the number of photons polarized along the \( y \) and \( x \) axes, \( \hat{S}_y \) is the analogue of \( \hat{S}_x \) in the basis rotated by \( 45^\circ \) and finally \( \hat{S}_z \) is the difference between the number of \( \sigma^+ \) and \( \sigma^- \) photons.\(^7\)

Using definitions (2.2.9) it is easy to show that the components of the Stokes vector obey an angular momentum type commutator

\[ [\hat{S}_r, \hat{S}_s] = i\epsilon_{rst} \hat{S}_t \] (2.2.10)

where \( \{r, s, t\} \in \{x, y, z\} \) and \( \hat{S}_0 \) commutes with all other components of \( \mathbf{S} \). The corresponding uncertainty relation is

\[ \text{var}(\hat{S}_r)\text{var}(\hat{S}_s) \geq \langle \hat{S}_t \rangle^2/4. \] (2.2.11)

For light polarized along \( x \), the **coherent polarization state** is defined by

\[ \text{var}(\hat{S}_y) = \text{var}(\hat{S}_z) = \frac{1}{2} |\langle \hat{S}_x \rangle| = \frac{1}{4} \langle \hat{N}_{\text{ph}} \rangle \equiv N_{\text{ph}}/4. \] (2.2.12)

\(^7\)The reason to use an unusual factor of \( 1/2 \) in the definitions Eqs. 2.2.9 will become clear soon.
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Note that by inserting an extra factor of 1/2 in the definitions of the Stokes vector components, Eq. 2.2.9, we have achieved a perfect symmetry between the description of light in this section and the description of the pseudo-spin in section 2.1.3.

2.3 QND measurement as a means of creating spin squeezing

The idea of producing spin squeezing by a QND measurement was first introduced in [4]. The feasibility of the required QND scheme was demonstrated in [25] and spin squeezing using this method was first reported in [26].

One of the systems considered in [4] consisted of an ensemble of particles each characterized by a spin-1/2 ground level and a spin-1/2 excited level resembling the letter X (Fig. 2.2). It was shown that for an off-resonant probe the dipole Hamiltonian describing the light-atom interaction can be written as

\[ \hat{H}_I = -\frac{4 g\alpha_0}{3\Delta} \hat{S}_z \hat{J}_z \]  \hspace{1cm} (2.3.1)

where \( \hat{J}_z \) and \( \hat{S}_z \) are the z-components of the ground state atomic spin and the

![Diagram](image-url)
Stokes vector, respectively, and where
\[ g = \frac{\omega}{2\epsilon_0 V} = \frac{\pi}{\epsilon_0 \lambda} \frac{1}{A\tau}, \]
\[ \alpha_0 = \frac{3\epsilon_0 \hbar \Gamma \lambda^3}{8\pi^2}. \]

Hence, the interaction Hamiltonian depends inversely on the probe detuning \( \Delta \), cross section area \( A \) and the pulse duration \( \tau \). The remaining constants are parameters of the atomic transition. Equation 2.3.1 has been derived using adiabatic elimination of the excited states which in turn requires that their populations be negligible. Another assumption is that the optical depth at the detuning \( \Delta \) is small, that is \( OD(\Delta) \ll 1 \) or \( e^{-OD(\Delta)} \approx 1 \).

The evolution of the components of the vector operator \( \hat{\mathbf{J}} \) can be determined from the Heisenberg equation of motion
\[ \frac{\partial \hat{f}_r}{\partial t} = \frac{1}{i\hbar} [\hat{f}_r, \hat{H}_I]. \] (2.3.3)

Substituting 2.3.1 for the commutator on the right-hand side yields
\[ \frac{\partial \hat{f}_x}{\partial t} = \frac{4g\alpha_0}{3\Delta\hbar} \hat{S}_z \hat{f}_y, \]
\[ \frac{\partial \hat{f}_y}{\partial t} = -\frac{4g\alpha_0}{3\Delta\hbar} \hat{S}_z \hat{f}_x, \] (2.3.4)
\[ \frac{\partial \hat{f}_z}{\partial t} = 0. \]

Similarly
\[ \frac{\partial \hat{S}_x}{\partial t} = \frac{4g\alpha_0}{3\Delta\hbar} \hat{S}_y \hat{f}_z, \]
\[ \frac{\partial \hat{S}_y}{\partial t} = -\frac{4g\alpha_0}{3\Delta\hbar} \hat{S}_x \hat{f}_z, \] (2.3.5)
\[ \frac{\partial \hat{S}_z}{\partial t} = 0. \]

Using these evolution equations we can write down input-output relations for the components of \( \hat{\mathbf{J}} \) and \( \hat{\mathbf{S}} \) up to first order in the interaction time \( \tau \)
\[ \hat{f}_x^{(\text{out})} = \hat{f}_x^{(\text{in})} + \frac{4g\alpha_0\tau}{3\Delta\hbar} \hat{S}_z \hat{f}_y, \]
\[ \hat{f}_y^{(\text{out})} = \hat{f}_y^{(\text{in})} - \frac{4g\alpha_0\tau}{3\Delta\hbar} \hat{S}_z \hat{f}_x, \] (2.3.6)
\[ \hat{f}_z^{(\text{out})} = \hat{f}_z^{(\text{in})}. \]
and

\[
\hat{S}_x^{\text{(out)}} = \hat{S}_x^{\text{(in)}} + \frac{4 \alpha_0 \tau}{3} \hat{S}_y \hat{f}_z, \\
\hat{S}_y^{\text{(out)}} = \hat{S}_y^{\text{(in)}} - \frac{4 \alpha_0 \tau}{3} \hat{S}_x \hat{f}_z, \\
\hat{S}_z^{\text{(out)}} = \hat{S}_z^{\text{(in)}},
\]

(2.3.7)

where the explicit dependence on \( \tau \) cancels out that of \( g \) (cf. Eq. 2.3.2).

If both light and atoms are polarized along the \( x \)-axis, we can replace the \( x \)-components, \( \hat{J}_x \) and \( \hat{S}_x \), by their average values. The remaining operators then transform as

\[
\hat{J}_y^{\text{(out)}} = \hat{J}_y^{\text{(in)}} - \frac{4 \alpha_0 \tau}{3} \Delta \hbar \langle J_x \rangle \hat{S}_z, \\
\hat{J}_z^{\text{(out)}} = \hat{J}_z^{\text{(in)}}, \\
\hat{S}_y^{\text{(out)}} = \hat{S}_y^{\text{(in)}} - \frac{4 \alpha_0 \tau}{3} \Delta \hbar \langle S_x \rangle \hat{J}_z, \\
\hat{S}_z^{\text{(out)}} = \hat{S}_z^{\text{(in)}}.
\]

(2.3.8)

The third one of equations 2.3.8 describes a rotation of \( \hat{S}_x \) onto \( \hat{S}_y \) by an amount that is proportional to \( \hat{J}_z \). This entangles the atomic and the light variables [27]. With both \( \hat{J} \) and \( \hat{S} \) initially prepared in coherent states pointing in the \( x \)-direction the average values of \( \hat{J}_z \) and \( \hat{S}_z \) are zero and their uncertainties obey 2.1.18 and 2.2.12, respectively. Equations 2.3.8 leave \( \hat{J}_z \) and \( \hat{S}_z \) unaffected while \( \hat{S}_y \) and \( \hat{J}_y \) acquire contributions from \( \hat{J}_z \) and \( \hat{S}_z \), respectively. Therefore, a (destructive) measurement of \( \hat{S}_y \) reduces the uncertainty of \( \hat{J}_z \) below the standard quantum limit given by 2.1.18. Simultaneously, because of the first one of Eqs. 2.3.8, the light variable \( \hat{S}_z \) is mapped onto the atomic variable \( \hat{J}_y \).

While the former of the two effects corresponds to spin squeezing, the latter can be utilized to implement a quantum memory for light [2].

### 2.3.1 Signal to noise ratio and degree of squeezing

We now compute the amount of squeezing implied by Eqs. 2.3.8. To that end we calculate the uncertainty of a measurement of \( \hat{S}_y \) according to the third one
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of Eqs. 2.3.8.

\[
\text{var}(S_y^{\text{out}}) = \text{var}(S_y^{\text{in}}) + \left(-\frac{4 g\alpha_0 \tau}{3 \Delta h}\right)^2 \langle S_x \rangle^2 \text{var}(J_z)
\]

\[
= \frac{N_{\text{ph}}}{4} + \left(\frac{4 g\alpha_0 \tau}{3 \Delta h}\right)^2 \frac{N_{\text{ph}}^2 N_{\text{at}}}{4}.
\]

The ratio of the two terms, which in this measurement plays the role of a signal to noise ratio, is given by

\[
\kappa^2 \equiv \left(\frac{4 g\alpha_0 \tau}{3 \Delta h}\right)^2 \frac{N_{\text{ph}} N_{\text{at}}}{4}.
\]

It can be shown [28, 29] that the degree of squeezing is expressed by

\[
\xi^2 = \frac{1}{1 + \kappa^2}.
\]

Substituting for \(g\) and \(\alpha_0\) from Eq. 2.3.2 we obtain

\[
\kappa^2 = N_{\text{at}} \frac{\lambda^2 / 2\pi}{A} N_{\text{ph}} \frac{\lambda^2 / 2\pi}{A} \left(\frac{\Gamma}{2\Delta}\right)^2
\]

\[
= \frac{\sigma_0}{A} N_{\text{ph}} \frac{\sigma_0}{A} \left(\frac{\Gamma}{2\Delta}\right)^2
\]

\[
= \text{OD}_0 \cdot \eta,
\]

where

\[
\sigma_0 = \frac{\lambda^2}{2\pi}
\]

is the resonant cross-section,

\[
\text{OD}_0 = N_{\text{at}} \frac{\sigma_0}{A}
\]

is the resonant optical depth and

\[
\eta = N_{\text{ph}} \frac{\sigma_0}{A} \left(\frac{\Gamma}{2\Delta}\right)^2
\]

stands for the integrated photon scattering rate (number of photons scattered per atom over a pulse).

2.4 Off-resonant dipole interaction in \(^{87}\text{Rb}\)

In this and the following sections we generalize the light-atom Hamiltonian 2.3.1 to the case of \(^{87}\text{Rb}\). We show that in the limit where the detuning \(\Delta\) is large in comparison with the hyperfine splitting of the excited state the effective Hamiltonian is of precisely the same type. We commence by presenting the properties of Rubidium.
2.4.1 $^{87}\text{Rb}$

Rubidium is an alkali metal and as such contains a single electron on the last shell. For chemists this implies high reactivity, for atomic physicists a relatively simple level structure. Rubidium is usually found as a mixture of two isotopes, $^{87}\text{Rb}$ and $^{85}\text{Rb}$. Strictly speaking only the second one is stable but because of the extremely long life time $^{87}\text{Rb}$ is effectively stable [30]. Our apparatus has been designed to work with $^{87}\text{Rb}$ and we focus our discussion on this isotope.

The ground state is the $5^2S_{1/2}$ state. Because it is an S-state we have for the orbital angular momentum, $L = 0$. Thus the total electron angular momentum $J = L + S$

$$J \in \{|L - S|, |L - S| + 1, \ldots, L + S - 1, L + S\}$$

$$= 1/2.$$ 

(2.4.1)

Similarly, the total angular momentum of the atom $F = J + I$. Since the nuclear angular momentum is $I = 3/2$ we have

$$F \in \{|J - I|, |J - I| + 1, \ldots, J + I - 1, J + I\}$$

$$= \{1, 2\}.$$ 

(2.4.2)

The spin-orbit coupling leads to the fine structure splitting which in the ground state of $^{87}\text{Rb}$ is absent because $J$ takes only one value, $J = 1/2$. The coupling between the total angular momentum of the electron and the angular momentum of the nucleus leads to the hyperfine structure. The hyperfine structure splitting of the $5^2S_{1/2}$ state is equal 6.83 GHz. More precise values are given in Fig. 2.3 together with a complete level diagram of $^{87}\text{Rb}$ and $^{85}\text{Rb}$.

The excited state is a P state and so $L = 1$. This orbital angular momentum combined with the electronic spin lead to two fine structure levels, $5^2P_{1/2}$ and $5^2P_{3/2}$ with $J \in \{1/2, 3/2\}$. Both are connected to the ground state $5^2S_{1/2}$ by optical dipole transitions at 795 nm and 780 nm, respectively. The two transitions have been named D1 and D2 and together form the so called D line.

Each of the two excited states undergoes a hyperfine splitting resulting in $f \in \{1, 2\}$ for the lower excited state and $f \in \{0, 1, 2, 3\}$ for the upper one. The separations between these levels are 813 MHz and 72 MHz, 156 MHz and 267 MHz, respectively.
Figure 2.3: Detailed level diagram of $^{87}$Rb and $^{85}$Rb. Courtesy of Dr. de Echaniz.
2.4.2 Interaction Hamiltonian

As shown in Appendix A, by using the technique of spherical tensor decomposition the interaction of a single atom with a single mode of electromagnetic field can be written down as a sum of three components (Eqs. A.0.35 and A.0.37). We wish to apply the formalism developed in Appendix A to the case of an atomic ensemble. This is indeed easily done and the resulting Hamiltonian is

\[ \hat{H}_I = \hat{H}_I^{(0)} + \hat{H}_I^{(1)} + \hat{H}_I^{(2)} \]  

(2.4.3)

where the three components on the right hand side are due to the scalar (rank-0), vectorial (rank-1) and tensorial (rank-2) components of the polarizability \( \hat{\alpha} \) and are given by

\[ \hat{H}_I^{(0)} = \frac{1}{3} g \alpha^{(0)} \hat{N}_{ph} \hat{N}_{at}, \]

\[ \hat{H}_I^{(1)} = g \alpha^{(1)} \hat{S}_z \hat{F}_z, \]

\[ \hat{H}_I^{(2)} = g \alpha^{(2)} \left[ \hat{S}_x (\hat{F}_x^2 - \hat{F}_y^2) + \hat{S}_y (\hat{F}_x \hat{F}_y + \hat{F}_y \hat{F}_x) + \hat{S}_0 (\hat{F}_z^2 - \frac{1}{3} \hat{F}_z^2) \right]. \]

(2.4.4)

![Figure 2.4: A simplified level scheme of \(^{87}\)Rb. The arrows show the probe configuration.](image)

In our experiment we are concern with the \( F = 1 \) level and a probe detuned from the D2 transition as indicated in Fig. 2.4. For this level we have three Zeeman substates with \( m \in \{-1, 0, 1\} \). This in general leads to a 3-dimensional Hilbert
However, because of symmetry, the effect of the $m_F = 0$ state on circularly polarized light is independent of the handedness. This allows us to ignore the state $m_F = 0$ which reduces the dimension of the Hilbert space to 2. We can now define a pseudo-spin according to

$$
\hat{J}_0 = \frac{1}{2} \hat{N}_{at},
\hat{J}_x = \frac{1}{2} (\hat{F}_x^2 - \hat{F}_y^2),
\hat{J}_y = \frac{1}{2} (\hat{F}_x \hat{F}_y + \hat{F}_y \hat{F}_x),
\hat{J}_z = \frac{1}{2} \hat{F}_z.
$$

(2.4.5)

It can be shown by direct substitution that so defined components of $\mathbf{J}$ indeed obey the angular momentum commutation relation $[\hat{J}_r, \hat{J}_s] = i \epsilon_{rst} \hat{J}_t$. Similarly, feeding the explicit matrix form of the components of $\hat{F}$

$$
\hat{F}_x = \frac{1}{\sqrt{2}} \sum_{k=1}^{N_{at}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}_k,
\hat{F}_y = \frac{1}{\sqrt{2}} \sum_{k=1}^{N_{at}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}_k,
\hat{F}_z = \sum_{k=1}^{N_{at}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}_k
$$

into 2.4.5 yields

$$
\hat{J}_x = \frac{1}{2} \sum_{k=1}^{N_{at}} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}_k,
\hat{J}_y = \frac{1}{2} \sum_{k=1}^{N_{at}} \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}_k,
\hat{J}_z = \frac{1}{2} \sum_{k=1}^{N_{at}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}_k.
$$

(2.4.6)
Note that the middle column and the middle row of each of these matrices contain only zeros. Note also that this feature is preserved for any linear combination of the above matrices. We can therefore cross out the middle column and the middle row of each matrix which leaves us with a system of 2-dimensional matrices

\[ \hat{J}_x = \frac{1}{2} \sum_{k=1}^{N_{\text{at}}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_k, \]
\[ \hat{J}_y = \frac{1}{2} \sum_{k=1}^{N_{\text{at}}} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}_k, \]
\[ \hat{J}_z = \frac{1}{2} \sum_{k=1}^{N_{\text{at}}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}_k. \]

But these are just the matrices representing a collection of spin-1/2 particles. Because atoms in the \(|F = 1, m = 0\rangle\) can be neglected, the matrix form of \(\hat{J}_0\) also becomes 2-dimensional

\[ \hat{J}_0 = \frac{1}{2} N_{\text{at}} = \frac{1}{2} \sum_{k=1}^{N_{\text{at}}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}_k, \]

where \(N_{\text{at}}\) is now the sum of only the atoms in the other two Zeeman states.

Another way to write Eqs. 2.4.9 and 2.4.8 is the following

\[ \hat{J}_0 = \sum_{k=1}^{N_{\text{at}}} \frac{1}{2} (|+\rangle \langle +| + |-\rangle \langle -|)_k, \]
\[ \hat{J}_x = \sum_{k=1}^{N_{\text{at}}} \frac{1}{2} (|+\rangle \langle -| + |-\rangle \langle +|)_k, \]
\[ \hat{J}_y = \sum_{k=1}^{N_{\text{at}}} \frac{-i}{2} (|+\rangle \langle -| - |-\rangle \langle +|)_k, \]
\[ \hat{J}_z = \sum_{k=1}^{N_{\text{at}}} \frac{1}{2} (|+\rangle \langle +| - |-\rangle \langle -|)_k. \]

Using these definitions we can rewrite the components of the Hamiltonian (2.4.4) in the following form

\[ \hat{H}_l^{(0)} = \frac{4}{3} g \alpha^{(0)} \hat{S}_0 \hat{J}_0, \]
\[ \hat{H}_l^{(1)} = 2 g \alpha^{(1)} \hat{S}_z \hat{J}_z, \]
\[ \hat{H}_l^{(2)} = 2 g \alpha^{(2)} (\hat{S}_x \hat{J}_x + \hat{S}_y \hat{J}_y + \frac{1}{3} \hat{S}_0 \hat{J}_0). \]
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Just as in Sec. 2.3, the Hamiltonian depends inversely on the probe detuning $\Delta$, cross section area $A$ and the pulse duration $\tau$.

2.4.3 Polarizability

The rank-0, rank-1 and rank-2 components of the polarizability tensor $\tilde{\alpha}$ are given by (see Eq. A.0.34 of App. A)

$$\alpha^{(0)} = (-1)^{2f} \sum_{f'} \alpha_{f,f'}^{(0)} \left( (2f - 1)\delta_{f-1}^{f'} + (2f + 1)\delta_{f}^{f'} + (2f + 3)\delta_{f+1}^{f'} \right),$$

$$\alpha^{(1)} = (-1)^{2f} \sum_{f'} \alpha_{f,f'}^{(1)} \left( -\frac{2f - 1}{f} \delta_{f-1}^{f'} - \frac{2f + 1}{f(f+1)} \delta_{f}^{f'} + \frac{2f + 3}{f+1} \delta_{f+1}^{f'} \right),$$

$$\alpha^{(2)} = (-1)^{2f} \sum_{f'} \alpha_{f,f'}^{(2)} \left( \frac{1}{f} \delta_{f-1}^{f'} - \frac{2f + 1}{f(f+1)} \delta_{f}^{f'} + \frac{1}{f+1} \delta_{f+1}^{f'} \right),$$

with

$$\alpha_{f,f'}^{(0)} = \alpha_0 \frac{\Delta_{f,f'}^2}{\Delta_{f,f'}^2 + \Delta_{f,f'}^2} (-1)^{j' + 2i} (2j' + 1) \left\{ \begin{array}{ccc} j' & f' & i \\ f & j & 1 \end{array} \right\}^2$$

and $\alpha_0 = \frac{3\epsilon_0 \hbar \lambda^3}{8\pi^2}$ which is the same as defined in Eq. 2.3.2. Setting $i = 3/2$, $j = 1/2$, $f = 1$, $j' = 3/2$ which correspond to the lower ground level yields

$$\begin{array}{c|ccc|c}
 f' & 3/2 & f' & 3/2 \\ 1 & 1/2 & 1 \\
\hline
 0 & -\frac{1}{2\sqrt{3}} & -\frac{1}{3}\alpha_0 \mathcal{D}_{1,0} \\
 1 & -\frac{\sqrt{3}}{6\sqrt{2}} & -\frac{5}{18}\alpha_0 \mathcal{D}_{1,1} \\
 2 & -\frac{1}{2\sqrt{6}} & -\frac{1}{6}\alpha_0 \mathcal{D}_{1,2} \\
 3 & 0 & 0 \\
\end{array}$$

where we have defined $\mathcal{D}_{f,f'} \equiv \frac{\Delta_{f,f'}}{\Delta_{f,f'}^2 + \Gamma^2/4}$. Substituting these values in Eqs. 2.4.12 yields

$$\alpha^{(0)} = \alpha_0 \left( -1/3 \mathcal{D}_{1,0} - 5/6 \mathcal{D}_{1,1} - 5/6 \mathcal{D}_{1,2} \right),$$

$$\alpha^{(1)} = \alpha_0 \left( 1/3 \mathcal{D}_{1,0} + 5/12 \mathcal{D}_{1,1} - 5/12 \mathcal{D}_{1,2} \right),$$

$$\alpha^{(2)} = \alpha_0 \left( -1/3 \mathcal{D}_{1,0} + 5/12 \mathcal{D}_{1,1} - 1/12 \mathcal{D}_{1,2} \right).$$

Figure 2.5 shows plots of the three components versus the probe detuning, $\Delta$, from $|f' = 0\rangle$ as defined in Fig. 2.4. By assuming that all three $\Delta_{f,f'} \gg \Gamma$ we
can replace the three dispersion profiles $\mathcal{D}_{f,f'}$ by the corresponding $1/\Delta_{f,f'}$. This assumption is indeed not a severe one. We are now going to make another one which is much stronger, that for every $f', \Delta_{f,f'} \gg \Delta_{\text{hfs}}^c$. In this far-off-resonant regime we can write $\Delta_{1,2} \approx \Delta_{1,1} \approx \Delta_{1,0} \equiv \Delta$. With this assumption the exact Eqs. 2.4.15 transform to
\begin{align*}
\alpha^{(0)} &= -\frac{2\alpha_0}{\Delta}, \\
\alpha^{(1)} &= \frac{1}{3} \frac{\alpha_0}{\Delta}, \\
\alpha^{(2)} &= 0.
\end{align*}

This is a remarkable result. It tells us that for sufficiently large detunings (no matter red or blue) the rank-2 component of the polarizability can be neglected. Note that this is a consequence of the symmetry and it has nothing to do with the explicit dependence of all three components on $\Delta$. In fact they all scale in the same way, inversely with $\Delta$. Based on this fact we expect that as $\Delta$ grows the ratios of the three components will tend to some constant values. This is indeed the case and from Eq. 2.4.16 these ratios are given by $\alpha^{(2)}/\alpha^{(1)} \to 0$ and $\alpha^{(1)}/\alpha^{(0)} \to -1/6$. It is the former of these two that allows us to use the formalism developed for the X system in section 2.3 with only minor modifications that we introduce below.

---

8Similar results are obtained for other alkalis [27]
The symmetry of the problem has yet another consequence. As is apparent from
the second equation in (2.4.15) for large detunings the contributions of the excited
states $f'_1 = 1$ and $f'_2=2$ cancel each other out so that in this regime $a^{(1)}$ is entirely
due to $f'_0 = 0$.

We now assume that $\Delta F,F' \gg \Delta_{hfs}$ is valid which means that the detuning must
be well above 250 MHz and concentrate on the vectorial part of the Hamiltonian
in Eq. 2.4.11. By comparison to Eq. 2.3.1 we see that all the expressions derived
for the X system will hold also in this case if we make the substitution
$-\frac{4}{3} g_{a0} \Delta \rightarrow 2 g a^{(1)} = \frac{2 g a_0}{3 \Delta}$. In particular the input-output relations (2.3.8) will now read

\begin{align}
\hat{j}_y^{(out)} &= \hat{j}_y^{(in)} + 2 g a^{(1)} \tau \langle J_x \rangle \hat{S}_z, \\
\hat{j}_z^{(out)} &= \hat{j}_z^{(in)}, \\
\hat{s}_y^{(out)} &= \hat{s}_y^{(in)} + 2 g a^{(1)} \tau \langle S_x \rangle \hat{J}_z, \\
\hat{s}_z^{(out)} &= \hat{s}_z^{(in)},
\end{align}

or in terms of $a_0$ and $\Delta$

\begin{align}
\hat{j}_y^{(out)} &= \hat{j}_y^{(in)} + \frac{2 g a_0 \tau}{3 \Delta h} \langle J_x \rangle \hat{S}_z, \\
\hat{j}_z^{(out)} &= \hat{j}_z^{(in)}, \\
\hat{s}_y^{(out)} &= \hat{s}_y^{(in)} + \frac{2 g a_0 \tau}{3 \Delta h} \langle S_x \rangle \hat{J}_z, \\
\hat{s}_z^{(out)} &= \hat{s}_z^{(in)}.
\end{align}

The degree of squeezing is still expressed by Eq. 2.3.11. By writing $\kappa^2 = OD_0 \cdot \eta$
(2.3.12) and using the expressions for the resonant optical depth $OD_0 = N_{at} \sigma_0$
(2.3.14) and $\eta = N_{ph} \sigma_0 \left( \frac{\Gamma}{\Delta} \right)^2$ (2.3.15) we obtain an effective cross-section

$$\sigma_0 = \frac{\lambda^2}{4\pi}. \quad (2.4.19)$$

Note that in terms of the parameters present in Eqs. 2.4.17 and 2.4.18 the coupling
constant is given by (cf. Eq. 2.3.10)

$$\kappa^2 = \left( 2 g a^{(1)} \tau \right)^2 \frac{N_{ph} N_{at}}{4} = \left( \frac{2 g a_0 \tau}{3 \Delta h} \right)^2 \frac{N_{ph} N_{at}}{4} \quad (2.4.20)$$

### 2.4.4 The role of noise and losses

Up to now we have considered an ideal situation in which the atoms where in a
perfect coherent spin state and the probe was in a perfect coherent polarization
state. No extra noise sources apart from the fundamental quantum uncertainties have been taken into account. Likewise, losses of atoms and possible decoherence of the atomic state have not been included.

By loss we mean a situation when an atom no longer interacts with the probe. This can be due to the atom being lost from the trap or going to a state which is dark for the probe. Losses reduce the signal. We talk about decoherence when an atom is transferred from the desired state into another state which is not dark for the probe. This not only reduces the signal but also introduces extra noise. Both processes are discussed in Ref. [31]. An extensive theoretical treatment of the problem of a generally noisy measurement by using covariance matrix techniques can also be found in Ref. [32]. The experimental issue of detecting spin squeezing in a two-pulse experiment under noisy conditions is discussed in Ref. [33] where a general criterion is derived.

A typical QND experiment consists of three phases, the state preparation, the measurement, and possibly a verification. Any imperfections in the preparation phase will reduce the signal and potentially introduce noise. State preparation is discussed in Chap. 4. The measurement itself, if performed in a non-optimized fashion or under unfavorable conditions, has a similar detrimental effect. We do not mean here the quantum back action of the very measurement but rather factors like a too long separation between the probe pulses allowing for decoherence or stray magnetic fields causing precession of the atomic spins. A technique and an apparatus to treat the latter problem have been designed and are described in Chap. 6.

2.4.5 The use of a dipole trap

So far we have only considered a system consisting of an atomic sample of a diameter $A$ and a top-hat beam of the same diameter. This ensures a uniform coupling across the sample and across the beam. In this simple situation the entire dependence of the coupling on the geometry is cast in a single parameter, the on-resonance optical depth, $OD_0$ (see Eq. 2.3.12). Uniform samples exist in reality. For example this is effectively the case in atomic beam experiments [26] or in vapor cell experiments [1–3] as well as in experiments performed with large collections of atoms prepared and released from a MOT and probed with
a relatively small beam [10, 34, 35]. The opposite configuration where the probe beam is larger than atomic sample is also utilized [9, 36–38]. In this last situation it is necessary to use an imaging system which selects only the portion of the beam that has interacted with atoms blocking the remaining part. The intensity of the beam may then be assumed constant across the sample.

As is apparent from Eq. 2.3.14 to maximize the optical depth the sample should be made thin and long. This is why an elongated sample obtainable in a dipole trap based on a gently focused single beam [11, 39, 40] is expected to be beneficial for spin squeezing experiments. In fact, an achievement of spin-squeezing on the clock transition in an ensemble of $^{87}$Rb in a dipole trap has recently been reported [12]. In another recent\footnote{In fact there have recently been two more reports on “spin-squeezing”. However, the term “spin” as used in Ref. [41] refers to external (position) rather than internal degrees of freedom. Ref. [42] uses the term “spin-squeezing” referring to a reduction of noise as compared to the noise of the initial state. This alone does not prove squeezing.}, successful experiment the interaction was enhanced by a cavity [13]. Unfortunately, the particular geometry of dipole-trap experiments lacks both of the simple properties mentioned above. Neither is the intensity of the beam constant nor is the atomic density uniform. It is therefore essential to match the probe beam to the sample.

The density profile of an atomic sample captured in a simple dipole trap follows the product of the light intensity (usually Gaussian) distribution times the exponential Boltzmann factor. It is therefore natural to expect that a Gaussian probe beam will be a good match.

The problem of the matching between the probe beam and the atomic sample and its effect on the light-atom coupling were recognized early and have been addressed theoretically in Refs. [14] and [43]. We discuss this point in the context of our experimental setup in Chap. 4. In an experiment, the effective coupling strength has to be determined by a direct measurement and below we describe a simple technique than can be employed to that end.

### 2.4.6 Macroscopic polarization rotation: population imbalance

Consider an ensemble of atoms prepared in a coherent state with the quasi-spin vector pointing along the $z$-axis. Note that according to Eq. 2.4.5 this is
alent to polarizing the "real" spin along the z-axis. It is observed from Eq. 2.4.10 that the desired state is either $|f = 1, m = 1\rangle$ or $|f = 1, m = -1\rangle$, both of which can be prepared with a circularly polarized pump beam.

Moreover, consider a probe beam prepared in the same way as for the QND measurements, i.e. in a coherent polarization state along the x-axis (this corresponds to a linearly polarized probe along the x-axis or along the y-axis.). The evolution of the Stokes vector is still governed by the third one of Eqs. 2.4.18 but since all the quantities of interest are now classical we can replace the operators by their average values

$$\langle S_y^{(out)} \rangle = \frac{2 g \alpha_0 \tau}{3 \Delta \hbar} \langle S_x \rangle \langle J_z \rangle.$$  \hspace{1cm} (2.4.21)

The angle of rotation of the Stokes vector is then expressed by

$$\theta_S = \frac{\langle S_y^{(out)} \rangle}{\langle S_x \rangle} = \frac{2 g \alpha_0 \tau}{3 \Delta \hbar} \langle J_z \rangle,$$  \hspace{1cm} (2.4.22)

and the plane of polarization is rotated by half this value

$$\theta_E = \frac{1}{2} \theta_S.$$  \hspace{1cm} (2.4.23)

By using 2.4.20 we can write

$$\kappa^2 = \left( \frac{2 g \alpha_0 \tau}{3 \Delta \hbar} \right)^2 \frac{N_{ph} N_{at}}{4} = \left( \theta_S \right)^2 \frac{N_{ph}}{N_{at}} = \left( \theta_E \right)^2 \frac{4N_{ph}}{N_{at}}.$$  \hspace{1cm} (2.4.24)

The last equations have been derived based on the assumption of uniform coupling and as such they do not apply in general. Note, however, that whether the $y$-component of the Stokes vector is a macroscopic quantity $\langle S_y \rangle$ like in Eq. 2.4.21 or a quantum-mechanical operator $\hat{S}_y$ as in Eq. 2.4.18 and preceding, its evolution is always the same. Therefore, if the coupling is inhomogeneous like in a dipole trap, the coupling constant will be affected in the same way in both the classical and quantum measurements. Hence, we can use the macroscopic polarization rotation to determine the effective coupling constant

$$\kappa_{\text{eff}}^2 = \left( \theta_E \right)^2 4N_{ph} / N_{at}.$$  \hspace{1cm} (2.4.25)
Plugging thus determined value to Eqs. 2.4.18 yields

\[ \hat{j}_y^{(out)} = \hat{j}_y^{(in)} + 2\kappa_{\text{eff}} \sqrt{\frac{1}{N_{\text{ph}}N_{\text{at}}}} \langle J_x \rangle \hat{S}_z, \]

\[ \hat{j}_z^{(out)} = \hat{j}_z^{(in)}, \]

\[ \hat{S}_y^{(out)} = \hat{S}_y^{(in)} + 2\kappa_{\text{eff}} \sqrt{\frac{1}{N_{\text{ph}}N_{\text{at}}}} \langle S_x \rangle \hat{j}_z, \]

\[ \hat{S}_z^{(out)} = \hat{S}_z^{(in)}. \]  

(2.4.26)

or

\[ \hat{j}_y^{(out)} = \hat{j}_y^{(in)} + \kappa_{\text{eff}} \sqrt{\frac{N_{\text{ph}}}{N_{\text{at}}}} \hat{S}_z, \]

\[ \hat{j}_z^{(out)} = \hat{j}_z^{(in)}, \]

\[ \hat{S}_y^{(out)} = \hat{S}_y^{(in)} + \kappa_{\text{eff}} \sqrt{\frac{N_{\text{at}}}{N_{\text{ph}}}} \hat{j}_z, \]

\[ \hat{S}_z^{(out)} = \hat{S}_z^{(in)}. \]  

(2.4.27)

The expected degree of spin squeezing will then be given by (cf. Eq. 2.3.11)

\[ \xi^2 = \frac{1}{1 + \kappa_{\text{eff}}^2}. \]  

(2.4.28)

It follows that in order to achieve detectable degree of squeezing it is essential to maximize \( \kappa_{\text{eff}}^2 \). Since \( \kappa_{\text{eff}}^2 \) scales linearly with both \( N_{\text{ph}} \) and \( N_{\text{at}} \) we optimize the trapping and loading sequence so that a large number of atoms, \( N_{\text{at}} \), is obtained. On the other hand, the number of photons, \( N_{\text{ph}} \), cannot be increased without limits because of the damage to the atomic state which scales linearly with it (see Eq. 2.3.15). We discuss this last issue in section 2.4.8.

The final remark is that the expressions of the present chapter assume a perfect state preparation. This includes both light and atoms and applies equally to the quantum and classical measurements. Fullfiling these demands requires that the optical elements all along the path of the probe beam leave its polarization intact and that the optical pumping is 100% efficient\(^{10}\). In practice neither one of these requirements is trivial to meet. We discuss the preparation of the probe beam

\(^{10}\)This includes the optical pumping into the coherent state with \( J \) along the \( x \)-axis, used in the QND measurements, as well as the optical pumping that creates a coherent state with \( J \) along the \( z \)-axis, employed for the macroscopic rotation measurements described in this section.
as well as the implementation of the optical pumping in Ch. 4. Measurements of the coupling constant implementing the technique presented in this section are reported in Sec. 8.2 of Ch. 8. In a situation where the expected degree of squeezing is small, detecting it may be a challenge and a general criterion is required. Such a criterion has indeed been derived in Ref. [33].

### 2.4.7 Macroscopic polarization rotation: axial B-field

The value of the coupling constant determined using the method described in the previous section is dependent upon the efficiency of the optical pumping process used to prepare the atomic state. This is good on the one hand as any QND measurements performed on the system also require atoms in an appropriately prepared initial state. However, the two states are not the same and consequently the required optical pumping is also different. It is therefore desirable to find a method of estimating the coupling strength using the atomic state directly used in QND measurements. One way of doing so could consist in coherently evolving the latter state into another state that is measurable in some way.

In this section it is argued that it is possible to estimate an upper limit for the coupling efficiency in a simple measurement. The method is sensitive only to the total number of atoms in the states \(|F = 1, m = -1\rangle\) and \(|F = 1, m = +1\rangle\) and as such does not provide complete information about the state. Nevertheless, it is useful because it provides a means to confirm or disprove the efficiency of the state preparation.

Using the definition of \(J_z\), Eq. 2.4.5, we can rewrite Eq. 2.4.23 as

\[
\theta_E = \frac{1}{3} \frac{g a_0 \tau}{\Delta h} \langle J_z \rangle \\
= \frac{1}{3} \frac{g a_0 \tau}{\Delta h} \frac{N_+ - N_-}{2} \\
= \frac{N_+}{2} \frac{1}{3} \frac{g a_0 \tau}{\Delta h} - \frac{N_-}{2} \frac{1}{3} \frac{g a_0 \tau}{\Delta h},
\]

(2.4.29)

where \(N_+\) and \(N_-\) are the populations of \(|F = 1, m = 1\rangle\) or \(|F = 1, m = -1\rangle\), respectively. The last representation provides a useful interpretation; the total polarization rotation is an algebraic sum of the rotations caused by the populations, \(N_+\) and \(N_-\).
Consider a magnetic field applied along the z-axis. Its effect will be to shift the states \( |m = \pm 1 \rangle \) by \( \delta = \pm g_F m_F \mu_B B \) (Fig. 2.4.29). For these two states we have \( g_F = \pm 1/2 \) resulting in \( \delta = \pm 0.70 \) [MHz/G], where \( B \) is expressed in Gauss. This Zeeman shift affects the two components of Eq. 2.4.29 differently depending in their sign

\[
\theta_E = \frac{N_+}{2} \frac{g \alpha_0 \tau}{\hbar} \frac{1}{\Delta + \delta} - \frac{N_-}{2} \frac{g \alpha_0 \tau}{\hbar} \frac{1}{\Delta - \delta}. \tag{2.4.30}
\]

For \( |\delta| \ll |\Delta| \) we have \( \frac{1}{\Delta \pm \delta} \approx \frac{1}{\Delta} \pm \frac{\delta}{\Delta^2} \). Using this approximation we obtain

\[
\theta_E = \frac{1}{3} \frac{g \alpha_0 \tau}{\hbar} \left( \frac{N_+ - N_-}{2} \frac{1}{\Delta} - \frac{N_+ + N_-}{2} \frac{\delta}{\Delta^2} \right). \tag{2.4.31}
\]

The first term in the parenthesis corresponds to rotation caused by population imbalance already discussed in the last section. The second term is the “standard” Faraday rotation caused by the applied magnetic field. This term is linear in \( B \) thanks to the off-resonance assumption \( |\delta| \ll |\Delta| \).

Taking the population of \( |F = 1, m = 0 \rangle \) to be zero we can set, \( N_+ + N_- = N_{at} \). Using Eq. 2.4.20, the last expression can then be transformed to yield

\[
\theta_E = \sqrt{\frac{\kappa_{\text{eff}}^2}{N_{at} N_{ph}}} \left( \frac{N_+ - N_-}{2} - \frac{N_{at} \delta}{2 \Delta} \right). \tag{2.4.32}
\]

This expression allows us to estimate the coupling constant, \( \kappa_{\text{eff}}^2 \), by measuring the rotation angle for two different magnetic field values \( B_1 \) and \( B_2 \), corresponding to Zeeman shifts \( \delta_1 \) and \( \delta_2 \). Denoting by \( \theta_{E,1} \) and \( \theta_{E,2} \) the respective rotation angles...
angles we obtain
\[ \kappa_{\text{eff}}^2 = \frac{N_{\text{ph}}}{N_{\text{at}}}(\theta_{E,2} - \theta_{E,1})^2 \left( \frac{2\Delta}{\delta_2 - \delta_1} \right)^2. \]  

(2.4.33)

Note that this expression is only dependent upon the total number of atoms in the \(|F = 1, m = -1\rangle\) and \(|F = 1, m = +1\rangle\) states and not upon the distribution of the populations between these two states. By comparing the coupling constant measured using the two techniques one can verify the efficiency of the optical pumping process. Measurements of the coupling constant that use the technique presented in this section are reported in Sec. 8.3 of Ch. 8.

### 2.4.8 Coupling constant versus state damage

It is interesting to spend a little time to analyze the dependence of the coupling constant, \(\kappa^2\), and hence the achievable degree of spin squeezing on the number of photons in the probe pulse, \(N_{\text{ph}}\), and the detuning\(^{11}\) \(\Delta\). As discussed in the preceding section, and is apparent from Eq. 2.4.20, \(\kappa^2\) scales linearly with \(N_{\text{ph}}\) and inversely with \(\Delta^2\). One is therefore tempted to think that the outcome of the experiments can be improved by simply using a strong probe. The flaw of this reasoning is that it neglects the amount of damage in the atomic state caused by the probe light via spontaneous emission. We can safely assume that this damage is proportional to the number of photons scattered per atom during the probe pulse, \(\eta\). This quantity, as given by 2.3.15, has the typical scaling \(\frac{1}{\Delta^2}\) which is due to the scattering probability. Being an integrated quantity that refers to the whole pulse it is also proportional to the number of photons. We have therefore reached a conclusion that both \(\kappa^2\) and \(\eta\) scale in the same way with \(\frac{N_{\text{ph}}}{\Delta^2}\) and their ratio is independent of the number of photons in the pulse and also the detuning\(^{12}\). This fact is best seen if we write \(\kappa^2\) as (cf. Eq. 2.3.12)

\[ \kappa^2 = OD_0 \cdot \eta. \]  

(2.4.34)

Because the on-resonance optical depth, \(OD_0\), is by definition independent of the detuning and also the photon number in a probe pulse, \(\kappa^2\) scales with these two quantities in exactly that same way as \(\eta\) does.

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\(^{11}\)In this section we consider a homogeneous coupling and write \(\kappa^2\) rather than \(\kappa_{\text{eff}}^2\). The result applies equally to both.

\(^{12}\)This is valid in the limit of large detunings where the rank-2 effects can be neglected.
Apart from $\Delta \gg \Delta_{hf}^{rs}$, the argument of this section requires that we work in the limit of weak excitation, i.e. $N_{ph}$ and $\Delta$ can only be varied in such a way that the population of the excited state remains small.

### 2.4.9 Application of $\alpha^{(2)}$ to atom number measurements

**Figure 2.7:** Like in Fig. 2.5 but magnified around $\Delta = +35$ MHz.

**Figure 2.8:** Like in Fig. 2.5 but magnified around $\Delta = +462$ MHz.

In this section we go beyond the limit of large-detuning that allowed us to neglect the rank-2 terms in the Hamiltonian (2.4.11). Figures 2.7 and 2.8 show the same polarizability traces as in Fig. 2.5 but magnified around two particular values of
the detuning, $\Delta = +35 \text{ MHz}$ and $\Delta = +462 \text{ MHz}$, respectively. At these two points the rank-1 term disappears and the polarization-dependent part of the Hamiltonian (2.4.11) can be expressed as

$$\hat{H}_I = 2g\alpha^{(2)}(\hat{S}_x \hat{J}_x + \hat{S}_y \hat{J}_y). \quad (2.4.35)$$

The former of the two points is not useful because it lies too close from resonance. However, the latter one can be utilized for an atom number measurement. For a circularly polarized probe and the pseudo-spin polarized along the $x$-axis, it describes a rotation of $\hat{S}_z$ onto $\hat{S}_y$ proportionally to $\hat{J}_x$ which in turn is proportional to the atom number.

This should prove useful because the result can be compared against the number of atoms measured by more conventional methods like e.g. recapture in the MOT combined with a fluorescence measurement. Note also that, unlike the second technique, this atom number measurement is state-sensitive and as such may be helpful in quantifying the efficiency of optical pumping. Moreover, because the required atomic state is identical to that used in the QND measurements, this measurement can be performed almost simultaneously with a QND measurement and on the same sample which may allow for cancelling any shot-to-shot fluctuations in the number of atoms trapped in the dipole trap.

This and other applications of the general Hamiltonian (2.4.11), including quantum cloning and quantum memory are discussed in detail in Ref. [44].
Chapter 3

Atom traps

The invention of laser cooling techniques has had a huge impact on atomic physics. The attainable temperatures turned out to be much lower than could be produced in cryogenic methods. One of the famous achievements of the field was the experimental demonstration of Bose-Einstein condensation [45], [46], [47], [48].

In the context of the light-atom interaction described in the last chapter, cold atoms provide the opportunity to relatively enhance the quantum effects by working with smaller atomic ensembles while maintaining a high value of the optical depth. This comes at a price of a more complicated setup which has to include the necessary infrastructure to cool and trap the atoms.

There exist two kinds of effects associated with the interaction of a laser radiation and atoms that can be exploited for trapping and cooling. The first effect is due to radiative pressure that appears when photons are resonantly absorbed and then spontaneously re-emitted. This is the basis of the optical molasses technique that combined with an inhomogeneous magnetic field forms a magneto-optical trap which has become a principle source of cold atoms in contemporary experiments.

The other effect is due to the shift of the atomic energy levels. Unlike the process of absorption-re-emission which is dissipative, this one is conservative and as such it facilitates trapping without destroying the quantum state of the atoms.

We start with a brief presentation of the mechanisms that lead to cooling and trapping in a magneto-optical trap. The technical details of the MOT used in our setup follow. In the second part of this chapter we summarize the physics of the dipole force and conclude with a detailed description of the dipole trap setup.
This presentation focuses only on the most important, basic concepts. The field of laser cooling and trapping is very well covered in literature. Some general texts include [49], [50], [51], [52]. Historical perspective is provided by the Nobel lectures by those awarded for their work on the subject [53], [54], [55]. Some atomic physics books devote entire chapters to the field of laser colling and trapping [56], [57].

### 3.1 Laser cooling and magneto-optical trap

#### 3.1.1 Doppler cooling, optical molasses

Doppler cooling exploits the fact that in the frame of an atom moving in the field of a laser beam the frequency of the laser is altered by the Doppler effect. The basic concept can be explained using the example of a two-level atom. Although most experiments use alkali metal atoms which have a multilevel structure, the optical transition that is used for cooling in this case is closed which justifies the two-level model.

![Diagram of a two-level atom](image)

**Figure 3.1:** Doppler cooling force on a 2-level atom. The plot in (b) corresponds to $\Delta = -\Gamma$.

Consider an atom moving along the z-axis in its positive sense with velocity $v = |v|$ and a laser beam propagating in the opposite sense, characterized by a wavevector, $k = |k|$. We assume that factors that could affect the spatial pattern

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$^1$[52] contains an extensive bibliography.
of spontaneous emission are not present and the beam is weak so that saturation effects are negligible. The laser frequency is such that its detuning from the atomic transition $|g\rangle \rightarrow |e\rangle$ is of the order of the natural linewidth $\Gamma$. The main effect of the light on the atoms in this regime is due to the resonant scattering and as such is of dissipative character. In the frame of the atom the laser frequency $\omega$ will be larger than the frequency measured in the laboratory frame by the Doppler shift, $kv$. If $\omega$ is lower than the atomic transition frequency $\omega_0$ then the Doppler shifted frequency will be closer to $\omega_0$ than the original frequency $\omega$ was. Therefore the atom will scatter more photons than a stationary atom would. Since the photons come from a well specified direction while the spontaneous emission is isotropic, on average there is a momentum transfer from the beam to the atom, the beam exerts a force on the atom.

Apart from momentum transfer, there is also energy transfer, the atom-photon collision is in this case inelastic. In the atom frame the energy of the spontaneously emitted photons has a Lorentzian profile centered at $\omega_0$. In the laboratory frame, the spontaneously emitted photons exhibit a Doppler shift but this time the Doppler shift averages to zero. On average this energy transfer is equal to $\hbar kv$ per scattering event.

Since the thermal motion is equally likely to be in the opposite direction, a second beam counter-propagating with respect to the first one is applied. The two forces cancel each other for a stationary atom but not for an atom in motion. This is depicted in Fig. 3.1. The centers of the two Lorentzian absorption profiles are shifted with respect to one another by $2kv$ and their algebraic sum produces a force that in the region $|kv| < \Gamma$ scales linearly with $v$ but has the opposite sign thus providing viscous damping. In this regime the resulting force [50]

$$F = \hbar k \frac{\Gamma}{2} \left[ \frac{I}{I_{sat}} \frac{1}{4(\Delta - kv)^2/\Gamma^2 + (1 + 2I/I_{sat})} \right] - \hbar k \frac{\Gamma}{2} \left[ \frac{I}{I_{sat}} \frac{1}{4(\Delta + kv)^2/\Gamma^2 + (1 + 2I/I_{sat})} \right]$$

(3.1.1)

can be approximated by

$$F \approx -\beta v$$

(3.1.2)

where the damping coefficient $\beta$ is given by

$$\beta = -4\hbar k^2 \frac{I}{I_{sat}} \frac{2\Delta/\Gamma}{(1 + 4\Delta^2/\Gamma^2 + 2I/I_{sat})^2}$$

(3.1.3)

---

2As e.g. would be the case in an optical cavity.
with $\Delta = \omega - \omega_0$ and the saturation intensity, $I_{\text{sat}} = \frac{2\pi^2 \hbar \Gamma}{3 \lambda^2}$.

Combining three pairs of counter-propagating red-detuned beams results in what is called an optical molasses in which all three components of the thermal motion are attenuated.

The mechanism of Doppler cooling provides damping of the mean velocity of the atom. However, the random direction of the recoil in the process of spontaneous emission increases the uncertainty of the atomic velocity which results in heating. These two effects lead to a steady state characterized by a characteristic temperature. The Doppler cooling limit is given by

$$k_B T_D = \frac{\hbar \Gamma}{2}$$

which for $^{87}\text{Rb}$ equals 144 $\mu$K.

Early experiments with optical molasses [58] showed that the attainable temperatures are as much as an order of magnitude lower than $T_D$. This surprising result stimulated theoretical work on other cooling mechanisms.

### 3.1.2 Sub-Doppler cooling

Several processes can reduce the velocity of atoms well below the Doppler limit. They generally are only valid for atoms that are already very cold and so require the Doppler cooling to be present beforehand. The simplest sub-Doppler mechanism occurs in a standing wave created by counter-propagating beams, both linearly but orthogonally polarized[59]. The two beams must be coherent with each other but this is not difficult to achieve as typically all the cooling beams are derived from the same laser. Another requirement is that the atoms possess an internal structure richer than that of the two-level model atom. The first ingredient allows for creation of a standing wave in which the polarization changes rapidly on a sub-wavelength distance. Atoms that move in such a polarization pattern experience a light shift that depends on the particular magnetic Zeeman state and are subject to optical pumping processes, see Fig. 3.2. The idea is that the atom adiabatically moves from a valley to the hill thus exchanging its kinetic energy for potential energy. While climbing the hill its probability of absorbing a photon increases and if an absorption happens then the atom has a chance of decaying into the other ground state which for the polarization in that particular...
place is a dark state and so the atom moves to another hill where the process repeats. Because the state into which the atom fell has a lower energy, there is a net energy loss and this energy comes from the atom thermal motion. In a 1D molasses setup the two polarizations are circular, not linear, which requires another model. It turns out that cooling occurs also in this configuration[59]. In a realistic 3D molasses a polarization pattern results which enables both cooling mechanisms. Detailed description of the various mechanisms of sub-Doppler cooling may be found in literature [60].

Extremely low temperatures are achieved in the process of evaporative cooling. The evaporation process selectively removes the atoms with highest velocities from the sample retaining the slower ones. Elastic collisions then lead to thermalization and the final temperature may be in the order of nK [61]. Evaporative cooling requires special facilities and does not occur in the MOT itself.

### 3.1.3 Trapping in a MOT

When a magnetic field of constant gradient is added to an appropriately polarized optical molasses a position dependent force results. This was first proposed in 1986 \(^3\) and demonstrated in [62]. The simplest example is provided by an atom with \(F = 0\) and \(F' = 1\) in one dimension. As a consequence of the applied

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\(^3\)The idea is usually attributed to Jean Dalibard [54].
magnetic field the degeneracy of the Zeeman sub-states of the excited level is removed everywhere except the very center, \( z = 0 \). Moreover, because the shift is proportional to the magnetic field, it follows the field gradient and the result is shown in Fig. 3.3. For \( z > 0 \) an atom in its ground state is more likely to absorb a \( \sigma^- \) photon and receive a momentum kick towards \( z = 0 \). The same argument applies to an atom located on \( z < 0 \). It, too, feels a restoring force towards the center, \( z = 0 \). The force on the atom is given by

\[
F = -\frac{\beta g_e \mu_B B'}{\hbar k} z
\]

(3.1.5)

where \( \beta \) is the damping coefficient, (Eq. 3.1.3), \( g_e \) is the g-factor of the excited level, \( \mu_B \) is the Bohr magneton and \( B' \) is the (constant) field gradient.

A three-dimensional trap is created by combining a three-dimensional molasses with a quadrupole field produced by a pair of anti-Helmholtz coils. In practice the trap may use 6 beams along the \( x, y, \) and \( z \) axis, 3 retro-reflected beams, 4 beams in a tetrahedral configuration, or just one beam with a pyramidal mirror [63].

The physics of the magneto-optical trap is rich and has been subject of many studies which among others include [64], [65], [66], [67].
3.2 The MOT

The MOT apparatus was originally built in Innsbruck and a detailed description can be found in Refs. [15] and [16]. At ICFO one of the laser sources has been replaced by a more powerful version which significantly improved the loading rate.

The design consists of two stages. The upper stage confines only in two dimensions letting the atoms move freely in the vertical direction. The main stage is located below and a weak vertical beam is used to aid the transfer between the two stages.

The background pressure in the main stage is below $10^{-10}$ mbar. It employs three retro-reflected beams with the fluorescence light collected by a high NA lens and split between a calibrated photodiode and a CCD camera. The loading rate measured in this setup is typically $10^7$ s$^{-1}$ and atom number density is approximately $10^{10}$ cm$^{-3}$. The attainable temperature, as inferred from the expansion of the cloud in a time of flight measurement, is about 20 $\mu$K.

The apparatus is controlled from a computer via a program based on NI Labview. The PC runs MS Windows and the input/output communication is implemented with NI cards. The best time resolution that can be achieved in practice with this system is 2 $\mu$s.

3.2.1 Lasers

All the beams for the MOT are produced with extended-cavity diode lasers. Light is delivered to the trap setup via single-mode polarization-maintaining fibers. The axial gradient in the main MOT is up to 35 G/cm with the radial gradient being half this value.

2D cooler

The cooling laser for the two-dimensional stage has been built at ICFO at an early stage of this project. This includes the laser heads and most of the electronic

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$^4$Cooling laser for the two-dimensional stage. See further in the text for details.

$^5$[68] is a nice review of the subject.
blocks like the current drivers, temperature controllers and PID locking circuits$^6$. In order to deliver high power a master-slave configuration [69] [70] is used. The external cavity of the master laser exploits the -1 diffraction order which is partially reflected on each side of the beam splitter.

![External cavity of the 2D cooler.](image)

**Figure 3.4:** External cavity of the 2D cooler.

This configuration has the disadvantage that a large fraction of power is lost but in the case of a master laser this is not critical. A very practical feature is that the two produced beams emerge under the same angle no matter the angle of the grating. This facilitates alignment of the external cavity without having to realign the produced beams. The frequency of the master laser is locked to the crossover signal of $F = 2 \rightarrow F' = 2$ and $F = 2 \rightarrow F' = 3$ transitions in a typical saturated absorption setup and then shifted to the cooling transition $F = 2 \rightarrow F' = 3$ minus one linewidth, with a double-pass acousto-optic modulator (AOM). The sub-Doppler resolution is obtained by means of saturated absorption and locking is accomplished by the frequency modulation technique$^7$. The power delivered by the slave laser depends only on the diode used and in our system is about 30 mW in the trap.

### 3D cooler

The cooling laser of the three-dimensional MOT has a Littman-Metcalf type extended cavity [72] and the frequency locking is accomplished using the dichroic atomic vapor laser lock (DAVLL) method. The main advantage of this technique is that it avoids modulating the current of the diode and so there are no sidebands introduced. As in the case of the 2D cooler, locking signal is the crossover

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$^6$Most of the designs have been provided by Dr de Echaniz and the ICFO electronic workshop has been very instrumental.

$^7$For a good introduction see [71]
of the $F = 2 \rightarrow F' = 2$ and $F = 2 \rightarrow F' = 3$ transitions. A small difference is that in this case the AOM is placed in the spectroscopy path and the laser is running at the frequency of the cooling transition. This saves power but it also makes the locking less reliable if fast change of the detuning is required.

Repumper

The cooling transitions are always closed transitions but the probability of exciting the atom into other upper level than $F' = 3$, although small, is non-zero. From $F' = 2$ or $F' = 1$ the atom can then decay to $F = 1$ and then it is lost for the cooling process. In practice, if no extra laser is implemented that brings the atoms accumulated in $F = 1$ back to $F = 2$, the cooling process stops very quickly. The repumper laser is also a Littman-type but locking is done using FM technique to the crossover signal of the $F = 1 \rightarrow F' = 1$ and $F = 1 \rightarrow F' = 2$ and the shift to the $F = 1 \rightarrow F' = 2$ is accomplished with an AOM.

### 3.3 Dipole potential, light shifts and scattering rate

Dipole trapping of atoms has common origins with optical tweezers. The idea goes back to Lethokov [73] and Ashkin [74] and it was first demonstrated by Bjorkholm et al. in 1978 [75]. In this section the main theoretical results are summarized. We start with a two-level atom interacting with off-resonant laser light treated classically. This basic description is then extended to include the case of multilevel atoms. The results of this semiclassical treatment are usually satisfactory, nevertheless for completeness the quantum view is also presented. Again, initially this is done for a two-level atom and the generalization to multilevel atoms and especially alkali metals follows. With a few exceptions the presentation develops along the line of reference [76].

Semiclassical treatment

Atoms in their ground state do not possess an electric dipole moment. However, a dipole moment can be induced, if an atom is placed an external electric field. This induced dipole will then interact with the same external field. In this section
we are concerned with atoms interacting with the AC-electric field of an off-resonant laser beam. In addition to the usual dipole approximation, we assume that the detuning of the laser from the atomic transition is much larger than the natural linewidth, \( \Delta \equiv \omega - \omega_0 \gg \Gamma \), and that scattering events are rare, \( R_{sc} \ll \Gamma \), which excludes saturation effects.

Under these assumptions, given a single classical mode
\[
E(r, t) = \hat{e}_\parallel \tilde{E}(r) \exp(-i\omega t) + c.c. \tag{3.3.1}
\]
and an atom with its two levels denoted by \( |g\rangle \) and \( |e\rangle \), the Hamiltonian of the system is
\[
\hat{H} = -\hat{d} \cdot \mathbf{E} \tag{3.3.2}
\]
It can be shown that in its ground state \( |g\rangle \) the atom experiences a potential
\[
U_{\text{dip}}(r) = -\frac{3\pi c^2}{2\omega_0^3} \left( \frac{\Gamma}{\omega_0 - \omega} + \frac{\Gamma}{\omega_0 + \omega} \right) I(r), \tag{3.3.3}
\]
where
\[
I = 2\varepsilon_0 c |\tilde{E}|^2. \tag{3.3.4}
\]
is the intensity. The relationship between the spontaneous emission rate \( \Gamma \) and the dipole matrix element can only be derived in a fully quantum model. It then turns out that
\[
\Gamma = \frac{\omega^3}{3\pi\varepsilon_0 \hbar c^3} |\langle e|\hat{d}|g\rangle|^2. \tag{3.3.5}
\]
Equation 3.3.3 clearly shows that trapping atoms with the dipole potential requires that the intensity of the beam be inhomogeneous. The same semiclassical treatment allows one to derive an expression for the scattering rate
\[
R_{sc}(r) = \frac{3\pi c^2}{2\hbar \omega_0^2} \left( \frac{\omega}{\omega_0} \right)^3 \left( \frac{\Gamma}{\omega_0 - \omega} + \frac{\Gamma}{\omega_0 + \omega} \right)^2 I(r). \tag{3.3.6}
\]
It is often, although not always, the case that the detuning is small in comparison with the transition frequency, \( \Delta \ll \omega_0 \). In this situation, the second term in Eqs. 3.3.3 and 3.3.6 can be neglected. Under this rotating-wave approximation, the expressions for the dipole potential and corresponding scattering rate simplify to
\[
U_{\text{dip}}(r) = -\frac{3\pi c^2}{2\omega_0^3} \frac{\Gamma}{\Delta} I(r), \tag{3.3.7}
\]
46
Both $U_{\text{dip}}$ and $R_{\text{sc}}$ scale proportionally with the intensity, however, they scale differently with the detuning. $U_{\text{dip}}$ follows the typical dispersive relation while $R_{\text{sc}}$ corresponds to the absorptive part and thus rolls off much faster. This suggests that in order to minimize scatter losses one has to use a large detuning. The price to pay is the required high intensity. Equation (3.3.7) also suggests that an attractive potential is created in the region of high intensity for negative $\Gamma$, a so called red-detuned trap, whereas for a blue detuning the region of maximum intensity will be repulsive. It is much easier to work with red-detuned traps because it is sufficient to use a single focused beam while for blue-detuning one has to use special techniques like for example a hollow beam.

This 2-level atom model can be generalized to include alkali metals. If the trapping light is detuned sufficiently far so that the hyperfine splitting of the excited states can be neglected we can use Eqs. 3.3.3 and 3.3.6 separately for each of the two excited fine structure levels $P_{1/2}$ and $P_{3/2}$ and then take a weighted sum of the two contributions [77]

$$U_{\text{dip}}(r) = -\frac{3\pi c^2}{2\hbar \omega_0^2} \left[ f_{D1} \frac{\Gamma_{D1}}{\omega_{D1}^3} \left( \frac{1}{\omega_{D1} - \omega} + \frac{1}{\omega_{D1} + \omega} \right) + f_{D2} \frac{\Gamma_{D2}}{\omega_{D2}^3} \left( \frac{1}{\omega_{D2} - \omega} + \frac{1}{\omega_{D2} + \omega} \right) \right] I(r),$$  \hspace{1cm} (3.3.9)

$$R_{\text{sc}}(r) = \frac{3\pi c^2}{2\hbar} \left[ f_{D1} \frac{\Gamma_{D1}^2 \omega^3}{\omega_{D1}^6} \left( \frac{1}{\omega_{D1} - \omega} + \frac{1}{\omega_{D1} + \omega} \right)^2 + f_{D2} \frac{\Gamma_{D2}^2 \omega^3}{\omega_{D2}^6} \left( \frac{1}{\omega_{D2} - \omega} + \frac{1}{\omega_{D2} + \omega} \right)^2 \right] I(r).$$  \hspace{1cm} (3.3.10)

The weights $f_{D1,D2}$ are the oscillator strengths of the D1 and D2 transitions, respectively. They are related to the spontaneous emission rates $\Gamma_{D1}$ and $\Gamma_{D2}$ by

$$\Gamma_{D1,D2} = \frac{e^2 \omega_0^2}{2\pi \epsilon_0 m_e c^3} \frac{2J + 1}{2J_{1,2}^* + 1} f_{D1,D2}. \hspace{1cm} (3.3.11)$$

Substituting the measured values of $\Gamma_{D1}$ and $\Gamma_{D2}$ we find $f_{D1} = 0.342$ and $f_{D2} = 0.696 [30]$. $\Gamma_{D1,D2}$ themselves are given by an expression similar to Eq. 3.3.5

$$\Gamma_{D1,D2} = \frac{\omega D1D2^3}{3\pi \epsilon_0 \hbar c^3} \sum_m |\langle e, m' | \hat{a} | g, m \rangle|^2. \hspace{1cm} (3.3.12)$$
Equations 3.3.9 and 3.3.10 have been intentionally left without making the rotating-wave approximation. One example when RWA cannot be applied is a trap based on a CO$_2$ laser, $\lambda = 10.6$ $\mu$m. The case of our dipole trap, $\lambda_{D1} = 795$ nm, $\lambda_{D2} = 780$ nm and $\lambda = 1030$ nm, is not so extreme, nevertheless RWA results in non-negligible errors as $(\omega_0 - \omega)/(\omega_0 + \omega) = 0.14$.

**Quantum description**

The fully quantum treatment requires that both atoms and light be described by quantum mechanical operators. The dipole Hamiltonian decomposes into three terms

$$\hat{H} = \hat{H}_A + \hat{H}_L + \hat{H}_{AL}, \quad (3.3.13)$$

with the atomic term given by

$$\hat{H}_A = \hbar \omega_0 |e\rangle \langle e|, \quad (3.3.14)$$

the laser mode described in terms of the creation and annihilation operators, $\hat{a}^\dagger$ and $\hat{a}$

$$\hat{H}_L = \hbar \omega \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right) \quad (3.3.15)$$

and the interaction part given by the Hamiltonian

$$\hat{H}_{AL} = -\hat{d} \cdot \hat{E}. \quad (3.3.16)$$

Applying second-order perturbation theory in the dressed atom picture, where light and the atom are considered as one quantum system, one finds that the energy levels of the combined system are shifted by the interaction term $\hat{H}_{AL}$ according to

$$\Delta E = \pm \frac{|\langle e|d|g\rangle|^2 |E|^2}{\Delta} = \pm \frac{3\pi c^2}{2\omega_0^3} \frac{\Gamma}{\Delta} I(r) \quad (3.3.17)$$

where the plus sign applies to the excited state and the minus sign to the ground state. This level shift is called a light shift or an AC Stark shift. The latter in analogy to the Stark effect, in which a perturbation in the form of a static electric field causes a similar shift. Note that the ground state shift is identical to the dipole potential derived classically (Eq. 3.3.7). For a red-detuned light atoms are trapped if they are in the ground state. Those in the excited state feel a repulsive potential.
Extension of the above results to the case of multilevel atoms requires replacing Eq. 3.3.17 with

\[
\Delta \varepsilon = \sum_{f',m' \neq f,m} \frac{|\langle f', m' | H_{\text{AL}} | f, m \rangle|^2}{\varepsilon_0 - \varepsilon_0'} \tag{3.3.18}
\]

where \( \varepsilon_0 \) and \( \varepsilon_0' \) are the eigenvalues of the unperturbed Hamiltonian and all relevant atomic states must be taken into account. Interaction Hamiltonian is still given by Eq. 3.3.16 but the calculation of the dipole moment, \( \hat{d} \), is more involved. As in Appendix A, going to the spherical basis and applying the Wigner-Eckart theorem allows us to factorize \( \langle f', m' | \hat{d}_q | f, m \rangle \) into a geometrical part and the “physical” component \( \langle j || \hat{d}_q || i \rangle \)

\[
\langle f', m' | \hat{d}_q | f, m \rangle = (-1)^{j'-m'} \begin{pmatrix} f' & 1 & f \\ -m' & q & m \end{pmatrix} \langle j' || \hat{d} || j \rangle \tag{3.3.19}
\]

The reduced matrix element \( \langle j' || \hat{d} || j \rangle \) does not depend on the magnetic numbers \( m \) and can be further reduced to

\[
\langle f' || \hat{d} || f \rangle = (-1)^{j'+i+f+1} \sqrt{(2j+1)(2j'+1)} \begin{pmatrix} j' & f' & i \\ j & f & 1 \end{pmatrix} \langle j'||\hat{d}||j \rangle \tag{3.3.20}
\]

The last reduced matrix element, \( \langle j' || \hat{d} || j \rangle \), is related to the spontaneous decay rate by

\[
|\langle j' || \hat{d} || j \rangle|^2 = (2j'+1) \frac{3\varepsilon_0 h \Gamma_j \lambda_0^3}{8\pi^2} = (2j'+1) \frac{3\pi\varepsilon_0 h c^3 \Gamma_j}{\omega_0^3}. \tag{3.3.21}
\]

The part of Eq. 3.3.19 that remains after factorizing out the reduced matrix element, \( \langle f' || \hat{d} || f \rangle \), is the so called transition strength. Its numerical value depends on the involved Zeeman states and on the polarization of light. It is often written down in terms of the Clebsch-Gordan coefficients \( C_{m,q}^{j} \)

\[
(-1)^{f'-m'} \begin{pmatrix} f' & 1 & f \\ -m' & q & m \end{pmatrix} = \frac{C_{m,q}^{j'}}{\sqrt{2j'+1}}. \tag{3.3.22}
\]

The dependence on the polarization is manifested by the presence of the index \( q \) (\( q = 0 \) for \( \pi \), \( q = +, - \) for \( \sigma^+ \) - polarized light, respectively). In the case of alkali atoms, it can be shown that if the detunings from the D1 and D2 lines are much larger than the hyperfine splitting, the potential can be written as [76]

\[
U_{\text{dip}}(r) = -\frac{\pi \varepsilon_0^2}{2} \left( \frac{\Gamma_{D1}}{\omega_{D1}^3} \frac{1 - q g_f m_f}{\Delta_{1,f}} + \frac{\Gamma_{D2}}{\omega_{D2}^3} \frac{2 + q g_f m_f}{\Delta_{2,f}} \right) I(r). \tag{3.3.23}
\]
Eq. 3.3.23 shows that by using circularly polarized light the trapping potential can be made state-dependent. In practice, with few exceptions [78], dipole traps are usually linearly polarized.

Although usually the terms due to the $P_{1/2}$ and $P_{3/2}$ are responsible for the vast amount of the total light shift, precise calculations must take into account coupling to higher lying states [79]. Interestingly, it has been shown that there exist wavelengths at which the shift of both excited and ground state are the same, hence resonance occurs at exactly the same frequency as for free atoms. Among other applications, these magic wavelengths can be employed for optical atomic clocks in an optical lattice [80].

### 3.4 The dipole trap

The dipole trap consists of a single, red detuned, Gaussian beam focused down to $w_0 = 56 \, \mu m$. The light in a single frequency mode, that is continuously monitored on a Fabry-Perot analyzing cavity, is delivered by a diode-pumped Yb:YAG solid state laser ELS Versa Disk. The wavelength is 1.03 $\mu m$ and the power achieved in the trap reaches 10 W with a typical value of 7 W. It can be adjusted by a combination of a $\lambda/2$ waveplate and a polarizing beam splitter. Additionally, a computer controlled acousto-optic modulator is used if fast adjustment is required.

The beam is spatially filtered and then coupled into a single-mode photonic band gap fiber, Crystal Technology Inc. Model 3110-125, that brings the trapping light into the MOT apparatus. The emerging Gaussian beam is collimated to a radius $w_0 = 0.45 \, \mathrm{mm}$. The trap is formed by focusing the collimated beam with an $f = 80 \, \mathrm{mm}$ achromatic lens, LINOS NIR doublet 322396 (L3 in Fig. 4.1 and Fig. 4.2). An achromat is required because the same lens is used to focus probe beams on the sample. A diagram of the optical setup, that shows also the dipole trap, can be found in Ch. 4 (Fig. 4.1). A detailed description of the trap design has been presented elsewhere [16].

The focused, $w_0 = 56 \, \mu m$, Gaussian beam provides an elongated potential with an FWHM aspect ratio of nearly 300 (Fig. 3.5 and Fig. 3.6) with resulting radial and axial frequencies of 790 Hz and 3.6 Hz, respectively. With 7 W in the trap
the potential depth computed using Eq. 3.3.9 and expressed in µK equals \[ \frac{U_0}{k_B} = 162 \text{ µK} \] while scatter losses are negligible. These parameters are computed below.

Figure 3.5: The potential of the dipole trap used in the experiment. Note the different scales along the transverse and the longitudinal directions.

### 3.4.1 The shape of the dipole potential

The intensity of a Gaussian beam carrying power \( P \) and propagating along the \( z \)-axis is described by (see e.g [81])

\[
I(\rho, z) = \frac{2P}{\pi w^2(z)} \exp \left( - \frac{2\rho^2}{w^2(z)} \right)
\]  

(3.4.1)

with the \( z \)-dependent waist, \( w(z) \)

\[
w(z) = w_0 \left( 1 + \frac{z^2}{z_0^2} \right)^{1/2}
\]  

(3.4.2)
Section 3.4.2 Trap depth and scattering rate

Substituting Eq. 3.4.1 for intensity in Eq. 3.3.9 and using

\[ -U_0 = -U(r) = \frac{3}{2} k_B T_{\text{depth}} \]  (3.4.6)

yields

\[ T_{\text{depth}} = 162 \, \mu K. \]  (3.4.7)

The last formula deserves a little attention. As a convention the trap depth is expressed using a different expression, \(-U_0 = k_B T_{\text{depth}}\). We do not follow this convention here because \(T_{\text{depth}}\) obtained using it does not correspond to any physical temperature. On the other hand, there exist a physical interpretation for Eq. 3.4.6. Consider a sample of atomic gas in free space (no potential). The average kinetic
energy per atom is given by the equipartition theorem \( \langle E_{\text{kin}} \rangle = \frac{3}{2} k_B T_{\text{depth}} \). At a certain instant a potential of depth \( U_0 \) is snapped on. Assume further that an atom whose kinetic energy is equal to the mean kinetic energy, \( \langle E_{\text{kin}} \rangle \), finds itself at the bottom of this potential. Clearly, this atom will only be trapped if

\[
\langle E_{\text{kin}} \rangle = \frac{3}{2} k_B T_{\text{depth}} < -U_0. \tag{3.4.8}
\]

The trap depth expressed in frequency units corresponds to the maximum of the ground state shift. This is equal to

\[
\Delta |_g \rangle = -5.07 \text{ MHz}.
\]

The shift of the excited state is positive and its magnitude is about two times larger [79]. Similarly, from Eq. 3.3.10 we find

\[
R_{\text{sc},0} = R_{\text{sc}}(r = 0) = 2\pi 0.17 \text{ s}^{-1}. \tag{3.4.9}
\]

Note also that if we had used the RWA, instead of the values quoted we would have obtained \( T_{\text{depth}} = 138 \mu\text{K} \) and \( R_{\text{sc},0} = 2\pi 0.28 \text{ s}^{-1} \).

### 3.4.3 Trap frequencies

For a mass \( m \) oscillating in a harmonic potential along \( \xi \) we have

\[
F(\xi) = -\frac{dU}{d\xi} = -mv^2\xi
\]

which allows us to find the oscillation frequency \( v \).

As can be seen from Eq. 3.3.9, the shape of the dipole potential mimics the shape of the trapping beam intensity. Hence, in order to determine the trap frequencies we have to show that the beam intensity profile can be approximated by a second order function of the space variables \( \rho \) and \( z \), and then differentiate the approximated expressions.

\[\text{Note that both } U \text{ and } U_0 \text{ are negative.}\]
Radial frequency

In the $z = 0$ plane, for $2(\rho^2 / w_0^2) \ll 1$, Eq. 3.4.1 can be approximated by the first two terms of its Taylor expansion

\[ I(\rho, 0) = \frac{2P}{\pi w_0^2} \exp\left(- \frac{2\rho^2}{w_0^2}\right) \]

\[ \approx \frac{2P}{\pi w_0^2} \left(1 - \frac{2\rho^2}{w_0^2}\right) \]

which is indeed of second order in $\rho$. The derivative is then given by

\[ \frac{dI(\rho, 0)}{d\rho} = -\frac{8P}{\pi w_0^2} \rho. \]  

(3.4.11)

(3.4.12)

Combining this with Eqs. 3.3.9 and Eq 3.4.10 yields

\[ \nu_{\rho, 0} = 2\pi 790 \text{ s}^{-1}. \]  

(3.4.13)

Axial frequency

In a similar fashion we expand Eq. 3.4.1 along the $z$-axis, assuming $z^2 / z_0^2 \ll 1$

\[ I(0, z) = \frac{2P}{\pi w_0^2} \frac{1}{1 + z^2 / z_0^2} \]

\[ \approx \frac{2P}{\pi w_0^2} \left(1 - \frac{z^2}{z_0^2}\right) \]

which, as required, is second order in $z$. Differentiating with respect to $z$ gives

\[ \frac{dI(0, z)}{dz} = -\frac{4P}{\pi w_0^2 z_0^2} z \]

(3.4.14)

(3.4.15)

and

\[ \nu_{z, 0} = 2\pi 3.6 \text{ s}^{-1}. \]  

(3.4.16)

3.4.4 Estimating the sample shape and the atom number

Under the assumption that atoms in the trap have thermalized, atom number density is given by the Boltzmann distribution

\[ n(z, \rho) = n_0 e^{-U(z, \rho) / k_B T}, \]  

(3.4.17)
with \( U(z, \rho) \) being the dipole potential. Figure 3.7 shows the contours that outline the \( n = n_0/2 \) region for various values of the temperature. The aspect ratio varies from \( 2z_{\text{FWHM}}/2\rho_{\text{FWHM}} = \frac{2 \times 2.6 \text{ mm}}{2 \times 10.8 \mu \text{m}} = 242 \) for \( k_B T = 0.1 \ U_0 \) to \( 2z_{\text{FWHM}}/2\rho_{\text{FWHM}} = \frac{2 \times 5.5 \text{ mm}}{2 \times 20.9 \mu \text{m}} = 261 \) for \( k_B T = 0.4 \ U_0 \). The effect of the exponential in Eq. 3.4.17 is thus to produce a sample whose linear dimensions are from 4 to 1.5 times smaller than the corresponding dimensions of the potential itself with the aspect ratio preserved to within 20\% (cf. Sec. 3.4.1).

Integrating Eq. 3.4.17 over space yields the total number of atoms

\[
N = \int_V n(z, \rho) dV. \tag{3.4.18}
\]

Fig. 3.8 shows the dependence of the atom number on the temperature for a range of peak density \( n_0 \) spanning some typical values. Unfortunately \( n_0 \) is not an easily measurable quantity.

A simple estimate of the number of atoms transferred to the dipole trap can be obtained by exploiting the energy conservation argument. An atom can only be captured if the sum of its kinetic energy and the potential energy due to the dipole trap at the moment when the dipole trap is switched on and all other trapping mechanisms (MOT, molasses) switched off, is smaller than 0. We assume that the atomic cloud to begin with is much larger than the effective volume of the dipole trap and is characterized by constant atom number density, \( n_{\text{MOT}} \), and by the Maxwell kinetic energy distribution with initial temperature, \( T_{\text{MOT}} \)

\[
f(E_k) = 2\sqrt{\frac{E_k}{\pi(k_B T_{\text{MOT}})^3}} e^{-E_k/k_B T_{\text{MOT}}}. \tag{3.4.19}
\]
Integrating this distribution over all kinetic energies that fulfill
\[
E_k + U \leq 0
\] (3.4.20)
and then over space gives an upper bound for the amount of atoms that can be transferred into the dipole trap
\[
N = n_{\text{MOT}} \int_V dV \int_{E_k=0}^{-U(z,\rho)} f(E_k) dE_k. \tag{3.4.21}
\]

The parameters \(n_{\text{MOT}}\) and \(T_{\text{MOT}}\) are readily measurable quantities. The energy distribution in the MOT is not Maxwellian [82] but can be assumed such as a first approximation. Thus estimated \(N\) is plotted in Fig. 3.9 for a range of \(n_{\text{MOT}}\) and \(T_{\text{MOT}}\) typical of our MOT [16]. The plot shows that the number of atoms grows with increasing \(n_{\text{MOT}}\) and decreasing \(T_{\text{MOT}}\). The latter dependence is much stronger and we are led to a conclusion that a well cooled sample is essential for an efficient loading. It should be noted that \(N\) calculated according to 3.4.21 is an upper bound rather than the actual number of atoms transferred because the method does not take into account losses in the transfer process. The next section describes the loading process from the an experimental perspective.
3.4.5 Dipole trap loading

Loading a dipole trap from a MOT is not quite trivial and has been studied as a subject in its own right [83]. Apart from a good geometrical overlap with the MOT, an efficient loading requires that the atoms are cold enough as compared to the depth of the dipole potential. Since our dipole trap is very long but has a very small cross-section another factor plays a critical role in loading large numbers of atoms. The MOT atoms must be compressed as much as possible in the region of the dipole trap. This compression is achieved by increasing the MOT magnetic field gradient. Decreasing the temperature also helps and can be done by going to the optical molasses regime with zero magnetic field. Since increasing the potential leads to heating while on the other hand the optical molasses provides no trapping which results in an expanding cloud of atoms, a compromise is necessary.

We experimentally find that for our trap the best results are obtained when the magnetic field gradient is increased and simultaneously the detuning of the cooling laser is increased. The magnetic field gradient is ramped up from 25 to 35 Gauss/cm in the axial direction while the detuning of the cooling laser is swept
from about -1.5 $\Gamma$ to -7 $\Gamma$. This is accompanied by ramping the intensity of both MOT lasers down. These changes are applied gradually and the whole process takes about 50 ms. At the end of this phase the magnetic field is suddenly switched off and the remaining small intensities of the MOT lasers are abruptly brought to zero with the repumping light being switched a couple of milliseconds earlier before the cooler in order to bring the atoms into the lower hyperfine state $F = 1$. This last step prevents spin changing collisions which constitute an important loss mechanism. Following this procedure results in collecting up to $5 \times 10^5$ atoms.

### 3.4.6 Atom detection and Imaging

Both number of trapped atoms and the spatial profile of the cloud are determined from the fluorescence signal acquired in a recapture phase just after the dipole trap is switched off. Setting the parameters back to the values required for the MOT takes several millisecond (magnetic field), the recapture phase itself is 100 ms long. The fluorescence light is collected on a high numerical aperture ($f/# = 1.8$) objective of focal length, $f = 36$ mm.\(^9\) The resulting collimated beam is then split on a cube that sends half of the beam intensity onto a calibrated photodiode while the other half is used for imaging. The latter is accomplished with monochrome CCD camera, COHU 4912-5010 equipped with an $f = 50$ mm objective. The fastest shutter time is $1/10000$ s. Magnification is given by the ratio of the two focal lengths $50$ mm/$36$ mm which together with the pixel size being 8.6 $\mu$m by 8.3 $\mu$m correspond to an area of about 6 by 6 $\mu$m being imaged on a single pixel.

### 3.4.7 Lifetime measurements

Lifetime of a dipole trap is usually limited by losses due to collisions between the trapped atoms and with background atoms. The rates of these two loss mechanisms scale differently with the number of trapped atoms. A phenomenological model proposed in [83] implies that the number of atoms in the trap, $N$, decays

\(^9\)The details of the optical design can be found in Ref. [16]
in time according to
\[
\frac{dN}{dt} = -\Gamma N - \beta' N^2,
\]
(3.4.22)

where \( \Gamma \) describes the decay due to the collisions with background particles and \( \beta' \) characterizes the loss due to inter-atomic collisions. The analytical solution has the form\(^{10}\)

\[
N(t) = \frac{N_0}{\exp(\Gamma t) + \frac{\beta'}{\Gamma} N_0 \exp(\Gamma t - 1)}
\]
(3.4.23)

In the limiting case of short time \( t \ll 1/\Gamma \) this reduces to

\[
N(t) \approx \frac{N_0}{1 + (\Gamma + \beta' N_0) t}
\]
(3.4.24)

which is a hyperbola. On the other hand, for storage times exceeding \( 1/\Gamma \), we can write

\[
N(t) \approx N_0 \frac{\Gamma}{\Gamma + N_0 \beta'} \exp(-\Gamma t)
\]
(3.4.25)

which is an exponent. Thus the model predicts a super-exponential decay at an early stage which gradually changes character to a simple exponential decay as the time goes on. In Fig. 3.10 experimental data are plotted along with a fit according to Eq. 3.4.23. As can be observed the fit is good except for very short and very long times. The excessive number of atoms at very short time (100 ms) is most likely due to atoms released from the MOT but not captured in the dipole.

\( ^{10} \)It can be easily obtained by solving the equation for \( t \).
CHAPTER 3: ATOM TRAPS

The 100 ms is enough for them to spread out of the field view of the imaging system. The disagreement between the data and the fit at very long times is a numerical artifact. The non-linear equation 3.4.23 makes the fitting procedure difficult. However, this portion of the data is important as it tells the exponential decay rate.

![Graph](image)

**Figure 3.11:** Last data points from Fig. 3.10 and an exponential fit according to Eq. 3.4.25.

In Fig. 3.11 we plot the last five data points\(^\text{11}\) in a logarithmic scale together with an exponential fit as given by Eq. 3.4.25. Thus extracted value of the exponential lifetime is

\[
\Gamma = 31 \pm 10^{-6} \text{ s.} \tag{3.4.26}
\]

The error bars drawn in Figs. 3.10 and 3.11 are given by the standard deviation of each of the data points and the uncertainties on the value of \(\Gamma\) in Eq. 3.4.26 inferred from the fitting procedure correspond to 95% confidence bounds.

### 3.4.8 Temperature measurements

In our experiment the temperature of atoms in the dipole trap is estimated using the time of flight (TOF) technique [58] [84]. Usually TOF is realized by shutting

\(^{11}\)For some measurement times there are more than one data point in the graph. In reality every point corresponds to an average over several experimental runs and what appears to be redundant data corresponds to series measured not consecutively but rather separated by an hour. The good agreement between points corresponding to the same trapping time indicates good repeatability.
off the trap, waiting for a specific time so that the atoms expand and then shining a resonant beam that gives an absorption image. Our approach is different in that we do not record an absorption profile but rather a fluorescence image.

Under thermal equilibrium not (Bose-) condensed particles in a parabolic potential form a Gaussian shaped cloud

$$\exp(-U/k_B T) \propto \exp(-r^2/2\sigma_r^2) \propto \exp(-U/k_B T) \propto \exp(-x^2/2\sigma_x^2) \exp(-y^2/2\sigma_y^2) \exp(-z^2/2\sigma_z^2). \quad (3.4.27)$$

If the potential is not spherically symmetric so is not the cloud

$$\exp(-U/k_B T) \propto \exp(-x^2/2\sigma_x^2) \exp(-y^2/2\sigma_y^2) \exp(-z^2/2\sigma_z^2). \quad (3.4.28)$$

The velocity distribution is given by the Maxwell-Boltzmann distribution which, too, is Gaussian

$$\exp(-\nu_x^2/2\sigma_v^2) \exp(-\nu_y^2/2\sigma_v^2) \exp(-\nu_z^2/2\sigma_v^2). \quad (3.4.29)$$

with $\sigma_v$ given by the equipartition theorem

$$\frac{m\sigma_v^2}{2} = \frac{k_B T}{2}. \quad (3.4.30)$$

Hence the total distribution is given by

$$f(x, y, z, \nu_x, \nu_y, \nu_z) \propto \exp(-x^2/2\sigma_x^2) \exp(-y^2/2\sigma_y^2) \exp(-z^2/2\sigma_z^2) \exp(-\nu_x^2/2\sigma_v^2) \exp(-\nu_y^2/2\sigma_v^2) \exp(-\nu_z^2/2\sigma_v^2). \quad (3.4.31)$$

If the time given to the atoms to fly freely after their release from the trap is much shorter than the mean time between collisions we can assume that the velocities are essentially constant during the whole TOF measurement and drop the velocity distribution part all together. This leaves us with just a distribution in coordinate space

$$F(x, y, z, \nu_x, \nu_y, \nu_z) \propto \exp(-x^2/2\sigma_x^2) \exp(-y^2/2\sigma_y^2) \exp(-z^2/2\sigma_z^2). \quad (3.4.32)$$

By the same assumption and taking $x$ to be the vertical direction we obtain

$$x = x_0 + v_{x0}t + \frac{gt^2}{2},$$
$$y = y_0 + v_{y0}t,$$
$$z = z_0 + v_{z0}t, \quad (3.4.33)$$
where \( v_{x_0}^2 = v_{y_0}^2 = v_{z_0}^2 = \sigma_v^2 \). Substituting these expressions in Eq. 3.4.32 and integrating over all possible initial positions yields

\[
F(x, y, z, t) \propto \exp\left(\frac{-(x - gt)^2}{2\sigma_x^2}\right) \exp\left(-\frac{y^2}{2\sigma_y^2}\right) \exp\left(-\frac{z^2}{2\sigma_z^2}\right),
\]  

(3.4.34)

where

\[
\begin{align*}
\sigma_x^2 &= \sigma_{x_0}^2 + \sigma_v^2 t^2 \\
\sigma_y^2 &= \sigma_{y_0}^2 + \sigma_v^2 t^2 \\
\sigma_z^2 &= \sigma_{z_0}^2 + \sigma_v^2 t^2.
\end{align*}
\]

(3.4.35)

Thus the spatial distribution remains Gaussian and the spread gives information about the temperature.

The elongated sample is not optically thick under radial observation and so it is reasonable to assume that the fluorescence signal is proportional to the local atom number density. To improve the signal to noise ratio, we integrate the signal along the longitudinal (z-) axis (Figs. 3.12 and 3.13). This is allowed as long as the potential can be approximated by a parabola which is to say for z smaller than the Rayleigh range (see Eq. 3.4.14).

For a sufficiently long time of flight the first line in Eq. 3.4.35 can be approximated by

\[
\sigma_x = \sqrt{\sigma_{x_0}^2 + \sigma_v^2 t^2} \approx \sigma_v t = \sqrt{\frac{k_B T}{m}} t.
\]

(3.4.36)

Motivated by this we fit a line to the measured values of the waist \( \sigma_x \) starting from TOF = 0.5 ms (Fig. 3.14). We find the temperature to be

\[
T = 24^{+7}_{-6} \mu K.
\]

(3.4.37)

The errors correspond to the uncertainties of the fit with 95% confidence bounds.
Figure 3.12: Fluorescence images of the atomic cloud after 0, 0.25, 0.50, 0.75, 1.00, 1.25 and 1.50 ms time of flight, respectively. The stripes that are visible in the images are an electronic artifact.
Figure 3.13: Vertical intensity profiles obtained by integrating images in Fig. 3.12 over the horizontal direction. Solid lines are Gaussian fits. The fact that some points appear to indicate negative intensities is due to background subtraction.
Figure 3.14: Vertical waist, $\sigma_x$, of the atomic cloud in Fig. 3.12 versus time of flight. The solid line is a linear fit to the waist determined from the Gaussians in Fig. 3.13 excluding TOF = 0 ms and TOF = 250 ms.
Chapter 3: Atom traps
Whether the initial state of the collective quasi-spin $\hat{J}$ is polarized along the $x$-axis as in the case of the QND measurements (Sec. 2.4.2 of Ch. 2) or along the $z$-axis (Sec. 2.4.6 of Ch. 2, Sec. 8.2 of Ch. 8), the required populations and coherences between the Zeeman substates are prepared in the process of optical pumping and efficiency of this process is important. QND measurements are particularly demanding in this respect because the required initial state is a coherent (quasi-) spin state. The purity of the polarization state of the probe light is just as important as that of the atomic state. This chapter presents the scheme used for optical pumping and describes how the quantum state of the probe beams is prepared. The details of the experimental setup are shown. The efficiency of the optical pumping in the current setup is discussed in chapter 8.

Optical pumping is a standard technique of preparing the quantum state of the atoms. It is based on interaction with resonant light of particular polarization which according to the selection rules induces only specific transitions [85]. Spontaneous emission completes the task bringing the excited atoms down to the ground level. The method has been pioneered by Kastler in the 50’s of the last century [86]. If the polarization of the pumping light is set accordingly, various distributions of the atomic populations and coherences can be produced in this way.

Preparing the quantum state of the probe amounts to setting its polarization as precisely as the quantum mechanical uncertainty allows. To that end two conditions must be met. First, a single frequency mode radiation with all classical intensity fluctuations well below the photonic shot noise is required. Second, the
polarization of this radiation must be filtered through a good polarizer. The first
demand is easily satisfied with contemporary external cavity lasers. As a polar-
izer one can use a Glan prism or a thin-film polarizing beam splitter. A more
formidable task than the production of an appropriately polarized probe is the
preservation of the polarization against spurious rotations as the probe propa-
gates through the dichroic optics necessary in our setup because of the collinear
geometry of the probe and the optical dipole trap.

Apart from being prepared in a correct polarization state the probe beam must
be mode-matched to the atomic sample in order to maximize the atom-light cou-
pling. This requires setting the probe waist to an optimum value and a correct
alignment. These problems are discussed in the last section of this chapter.

### 4.1 Optical pumping

According to the first one of Eqs. 2.4.8 a coherent quasi-spin state polarized along
the \( x \)-axis corresponds to a coherent superposition of states \( |F = 1, m = 1\rangle \) and
\( |F = 1, m = -1\rangle \) with equal probability for each of them. With quantization axis
defined to be the \( z \)-axis (Fig. 4.1), such a state is produced if the pumping light is
polarized along \( x \) or along \( y \). The pumping beam can then illuminate the atoms
from any of the three principal directions.

There are two reasons for not using the \( z \)-axis as a propagation axis of the pump
beam. First, the optical dipole beam, the probe beam and the circular pumping/auxiliary probe beam\(^1\) propagate along this direction. Adding one more
beam would complicate the setup. Second, the optical depth of the atomic sam-
ple is about two orders of magnitude larger along its symmetry axis (\( z \)-axis).
Therefore, pumping along this axis could be considerably slower. This leaves
the choice of either the \( x \)- or the \( y \)-axis. Because the imaging system is mounted
along the latter, it has been decided to install the pumping beam optics below
the atomic sample so that the beam propagates along the \( x \)-axis illuminating the
atoms from below.

Figure 4.2 shows the arrangement in the \( xz \)-plane. This, like also all other pump
and probe beams, is delivered to the apparatus via a single mode optical fiber.

\(^1\)Description of the circular pumping beam and the auxiliary probe beam follows.
Figure 4.1: Simplified diagram showing the arrangement of the beams, the imaging and the detection setup. Top view.

Figure 4.2: Simplified diagram of the optical pumping optics. Side view.
The plano-concave cylindrical lens L6, $f = -80$ mm, expands the beam so that its initial diameter, $2w_0 = 2.0$ mm increases along one of the axes to $2w_0 = 14.1$ mm at the sample. A large beam is used in order to illuminate the sample uniformly. In order to preserve the polarization, a gold mirror, GM2, is used. The quarter waveplate, HW4, and the half waveplate, QW3, are placed in the beam in order to correct possible deviations of the polarization.

The linearly polarized pumping light is tuned to the $|F = 1\rangle \rightarrow |F' = 1\rangle$ transition and it is derived from the repumping laser of the MOT with an AOM. The efficiency of this optical pumping scheme could be greatly improved if an additional laser were installed and tuned to the $|F = 2\rangle \rightarrow |F' = 1\rangle$ or the $|F = 2\rangle \rightarrow |F' = 2\rangle$ transitions. This laser would bring the atoms from the $|F = 2\rangle$ back to the $|F = 1\rangle$ state. Without it there is always a non-negligible fraction of atoms that end up in the $|F = 2\rangle$ state and are lost for the experiment. Ideally, this repumper light would be unpolarized avoiding in this way any dark states in $|F = 2\rangle$. The cooling laser of the MOT is an ideal candidate and the necessary modifications in the setup are being implemented.

### 4.1.1 Circular optical pumping

The polarization rotation measurement proposed in Sec. 2.4.6 of Ch. 2 requires an initial state of the quasi-spin polarized along the $z$-axis or what is equivalent, the atoms must be pumped either into $|F = 1, m = -1\rangle$ or $|F = 1, m = 1\rangle$. For this a $\sigma^-$- or $\sigma^+$-polarized pumping light is required. We implement it with a $|F = 1\rangle \rightarrow |F' = 1\rangle$ tuned beam propagating along the $z$-axis. The size of the beam, $w_0$, is set between 60 and 100 $\mu$m at the trap center which is much larger than the atomic sample (Sec. 3.4.4 of Ch. 3) in order to provide a homogeneous intensity across the sample. This circular pumping beam passes through a half waveplate, HW3, and a quarter waveplate, QW2, (Fig. 4.1) which are set in such a way as to make the beam circularly polarized where it traverses the atomic sample.

Setting the right angles of waveplates HW3 and QW2 would be easy if not for the presence of dichroic elements used to combine (DC1) with and then again separate (DC2 and DC3) the circular pumping/auxiliary probe and the main (precision) probe beams from the optical dipole beam. These elements are based on
multilayer dielectric coatings and behave differently in interaction with s- and p-polarized light. Since the polarization of the auxiliary pump should be set circular at the atoms where it cannot be measured, a viable method consists in optimizing the polarization state in front of L4. The results are discussed in Sec. 8.2 of Ch. 8

4.2 Probe beams

The probe beam that is intended to be used in the spin-squeezing experiments (Sec. 2.4 of Ch. 2) and in the polarization rotation measurements of Sec. 2.4.6 and Sec. 2.4.7 (Ch. 2) is marked as “precision probe” in Fig. 4.1. The source used is a commercial extended cavity diode laser, TOPTICA DC100. Its frequency is locked to that of the MOT repumper laser and shifted using a computer controlled electronic offset-lock [87] on the basis of a digital PLL (AD4252) which will be described in greater detail elsewhere [79].

The input signal to the PLL is a beat note obtained by overlapping a portion of the beam of the laser to be locked with a reference laser beam on a fast photodiode. The PLL then provides an error signal that is fed back into the piezo crystal of the grating in the external cavity through a PID controller (low-frequency part) and to the current of the laser diode through a FET transistor (high-frequency). Apart from providing locking, this configuration allows for the detuning, $\Delta$, from the $|F = 1\rangle \rightarrow |F' = 0\rangle$ transition (Fig. 2.4) to be varied continuously from 150 MHz up to 3 GHz. An acousto-optic modulator is used to produce pulses as short as 300 ns.

The probe beam is guided through a thin-film polarizer (PBS2 in Fig. 4.1) of extinction ratio of $10^5$ (LINOS) in order to prepare a pure linear polarization state. As explained in the previous section, the common optics of the probe/pump beams and the dipole beam may introduce unwanted rotations to the polarization state. In order to correct for these effects a pair of waveplates (HW2 and QW1) has been introduced. The purity of the polarization state is confirmed in the measurements of Ch. 5 where poissonian scaling of the noise is demonstrated.

The waist of the probe beam is adjusted to $w_0 = 20 \mu m$ at the trap center.
justification for this particular size of the beam is given below.

4.2.1 Auxiliary probe

The atom number measurement discussed in Sec. 2.4.9 of Ch. 2 requires a circularly polarized probe. Since this auxiliary probe and the circular optical pumping beam (Sec. 4.1.1) are not used simultaneously, they share the optical path and are combined with the precision probe and the trapping beam on a 50/50 beam splitter (BS1). As in the case of the precision probe, pulses are produced with an AOM. The waist of the auxiliary probe beam is set to \( w_0 = 20 \mu m \) at the trap center, the same value as the precision probe.

4.3 Probe-sample mode matching

A proper matching of the probe beam to the atomic sample is essential. To appreciate this statement it is enough to remind ourselves about the size of the sample. It is a few millimeters long but only a few tens of \( \mu m \) wide (Sec. 3.4.4 of Ch. 3). A probe beam whose size is too small with respect to the size of the sample will miss those of the atoms that sit at the edges of the sample. This reduces the available signal. On the other hand, a probe that is too large, although not missing any atoms, will contribute extra noise from the portion of the beam that has not interacted with the sample. The net result is the same in both cases, a reduced signal to noise ratio. It is clear that a misalignment has a similar detrimental effect.

4.3.1 Optimum probe size estimate

In order to estimate an appropriate size of the probe beam we perform a simplified analysis. We start out with the noise contributed by light and assume that the only noise present is the photon shot noise whose standard deviation is proportional to the square root of the number of photons in the pulse. We then split the sample into small volume elements, \( dV \), and assume that the signal it contributes is proportional to the product of the atom number density and the probe


\[ dS \propto n(z, \rho) I(z, \rho) dV. \]  

(4.3.1)

Integrating over space yields the total signal. Taking the ratio of so calculated signal and noise yields a (unnormalized) signal to noise ratio. Figure 4.3 shows this

\[ \text{Figure 4.3: Probe-sample matching efficiency versus probe waists for various values of the temperature. (Each of the traces normalized individually.)} \]

SNR for various values of the temperature. Each of the traces has been normalized individually. As \( w_0 \) is varied the quantity that is kept constant is the peak intensity, \( I(0, 0) \). Another option is to keep the number of photons per pulse constant, however, the first choice is more adequate since the maximum intensity is limited by other factors like e.g. the maximum acceptable scatter rate. The traces show that for \( k_B T = 0.1 U_0 = 16 \, \mu K \) the optimum SNR is reached at \( w_0 = 33 \, \mu m \) and this value increases as the temperature is increased.

The main factor making the predictions of this model imprecise is the fact that it does not account for the wave nature of the light in the probe beam. A better model has been elaborated by Müller et al. ([14]). It treats the interaction as a coherent scattering process of the probe from the atomic sample. The interference of the original wave and the scattered waves then leads to an output beam that is measured by the photodetector. This analysis predicts that the optimum size is smaller than that predicted by the simplified model presented here and for our
Chapter 4: Atomic and Probe State Preparation

The trap is approximately $w_0 = 20 \, \mu\text{m}$. This is the reason why the probe size has been chosen to this value.

4.3.2 Probe alignment

Because of the specific elongated shape of the sample a precise alignment is necessary. Two methods can be employed here. The first one is the geometrical alignment of the probe beam to the trapping beam. The second one requires some signal from the atoms and it still uses a geometrical pre-alignment. The best candidate for the atomic signal is the macroscopic polarization rotation signal as described in Sec. 2.4.6 and Sec. 2.4.7 of Ch. 2 and measured in Ch. 8.

Although very attractive, alignment to an atomic signal is complicated in our setup by the fact that the position of the dipole trap beam is not adjustable. One then has to align the probe to the fixed dipole beam. This is in a sense good because the overlap of the MOT and the dipole trap is not affected. However, during the alignment the probe signal tends to walk off the active area of the photodiodes and one is never sure whether the observed change in the signal amplitude is due to a modified probe-sample overlap or the probe being misaligned on the photodetector. In principle a detector with large photodiodes could be used here but large photodiodes tend to be noisy and an appropriate bandwidth limitation would have to be implemented. This is a possible solution to the problem but requires developing a special type of detector just for the alignment. Instead, we have used a different method also based on a signal obtained from atoms.

By shutting the MOT repumper off a short time later after the cooling light, it is possible to pump the atoms into the hyperfine state, $F = 2$. If such prepared sample is illuminated by light tuned to the closed transition $F = 2 \rightarrow F' = 3$, atoms are efficiently removed from the region where the beam overlaps with the sample. This effect can be used as a diagnostic if the “blow-away” light is coupled into the probe fiber. Atoms in the region of weak overlap are not blown away. The fluorescence signal that they produce in a subsequent MOT recapture phase provides information about the overlap. This method works, however, we have not found the results obtained in this way to be any better than those achieved by careful geometrical alignment.

To maximize the efficiency of the geometrical alignment we overlap the probe
and the trap beams after a long distance, 16 m. Because of tiny errors in setting the distance between the focusing and the collimating lenses (L3 and L4 in Fig. 4.1), some little dichroism in the setup and other imperfections, the two beams diverge slightly faster than predicted by the Gaussian model. Large beams are not convenient to align as it is difficult to determine the center of the beams. To remedy this to some degree we have introduced adjustable irises in the path of the beams at experimentally determined positions. The size of the irises has also been experimentally adjusted so as to give a clear Airy pattern for both beams. When the irises have been correctly centered on the beams it has been possible to use the two resulting Airy patterns to improve the relative alignment of the two beams.

Figure 4.4: Overlapping Airy patterns of the probe and trap beams obtained when the two beams have been well aligned. For more information see text.
Detection

In the previous chapter the probe beam setup has been described. The quantity of interest that changes as an effect of the atom-light interaction is the probe polarization (Ch. 2). This change may be very small and therefore an efficient detection is of fundamental importance.

In this chapter construction of a balanced photo-detector is described. We start with a justification of the balanced detection technique. We then summarize the main characteristics of the photodiode as a photo-sensor and briefly discuss the basic electronic designs of photodetectors. Next, the detector circuit is introduced and its performance is discussed in detail with special emphasis on the applications where the limit is given by the photon shot noise. A simple data analysis algorithm has been devised which is required when the detector is employed for measurements of single polarity signals. The algorithm is described later, in Ch. 8.

5.1 Balanced detection

We are interested in an experimental situation outlined in Fig. 5.1. A linearly polarized probe beam is prepared so that the electric vector points along either of the two axes, $x$ or $y$. These polarization states correspond to the Stokes vector respectively aligned parallel or anti-parallel to $x$. Consider the case depicted in Fig. 5.1. The task consists in measuring $E_x$ or equivalently the angle of rotation $\theta$ that the polarization experiences due to interaction with atoms. The rotation is assumed to be small, $\theta \ll 1$. 

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The simplest way to measure $E_x$ is to separate it with a polarizing beam splitter. The amplitudes of the two signals will then be given by

\begin{align}
E_x &= -E_0 \sin \theta \approx -\theta E_0 \\
E_y &= E_0 \cos \theta \approx E_0
\end{align}

and consequently a photodetector placed in the path of the first beam will give a signal

$$P_x \propto \theta^2 |E_0|^2.$$ (5.1.2)

The light shot noise will also be detected to yield

$$\sqrt{\text{var}P_x} \propto \sqrt{P_x} \propto |\theta||E_0|,$$ (5.1.3)

where, for simplicity, bandwidth and other multiplicative factors have been omitted. Combining the last two expressions a power signal to noise ratio is obtained

$$\text{SNR} = \frac{P_x}{\sqrt{\text{var}P_x}} = |\theta||E_0|.$$ (5.1.4)

We now argue that the simple method described above is not optimal and introduce a balanced detection technique. The balanced measurement is accomplished in a basis rotated by $45^\circ$. Decomposing the electric field vector in this basis gives

\begin{align}
E_{\pi/4} &= E_0 \cos(\pi/4 + \theta) \approx \frac{1}{\sqrt{2}}(1 - \theta)E_0, \\
E_{3\pi/4} &= E_0 \sin(\pi/4 + \theta) \approx \frac{1}{\sqrt{2}}(1 + \theta)E_0.
\end{align}$
In practice this decomposition can be accomplished by using a polarizing beam splitter at 45° or more conveniently in a setup consisting of a half-wave plate at 22.5° and a polarizing beam splitter in a more conventional position. It follows from Eq. 5.1.5 that the difference of the powers in the two beams is given by

\[ P_{3\pi/4} - P_{\pi/4} \propto |E_{3\pi/4}|^2 - |E_{\pi/4}|^2 = 2\theta |E_0|^2 \]  

(5.1.6)

and the noise of this difference can be written as

\[ \sqrt{\text{var}P_{3\pi/4} + \text{var}P_{\pi/4}} = \sqrt{\text{var}P_0} \propto P_0 \propto |E_0|^2. \]  

(5.1.7)

Consequently the signal to noise ratio equals

\[ \text{SNR} = \frac{P_{3\pi/4} - P_{\pi/4}}{\sqrt{\text{var}P_{3\pi/4} + \text{var}P_{\pi/4}}} = 2\theta |E_0|. \]  

(5.1.8)

The last equations closely resemble those describing homodyne detection. Indeed, balanced detection can be thought of as a kind of self-homodyning. Bearing this in mind the following characteristics are not surprising.

Comparing the expressions derived for the direct and balanced measurements the following can be concluded. The signal obtained in a balanced configuration is \(2/|\theta|\) stronger than the directly detected. At the same time the noise is also amplified by a factor \(1/|\theta|\). This results in a signal to noise ratio better by a factor of 2 in the case of balanced detection. There are, however, two other more important differences. First, the sign of the balanced signal is that of the rotation angle whereas for the direct measurement this information is lost. Secondly, in any real detector the electronic noise will further reduce the SNR. It is here where the \(2/|\theta|\) (or \(1/|\theta|\) for the photon noise) gain becomes important making shot-noise limited measurements of weak signals possible.

These arguments are classical but they are sufficient to demonstrate the essence of balanced detection. Its principal drawback is the requirement of nearly perfect balancing. If this is not fulfilled any (classical) intensity noise will obscure the measured signal.

One could, in principle, devise yet another method of measuring polarization rotation. The rotation may be thought of as a result of the two circular components of the initial linear polarization experiencing different indices of refraction. The resulting phase shift can be measured in a double homodyne setup. This method is analyzed in [88].
5.2 Photodiode

The photodiode, like most other diodes, consists of a $p$-$n$ junction. The distinctive feature of the photodiode is that the junction is left exposed and thus can be illuminated with external light. At the wavelength of interest the best semiconductor material for a photodiode is silicon (Si). The treatment in this section closely follows the superb introduction from Hamamatsu [89].

Because the concentration of electrons in the $n$-type semiconductor is greater than in the $P$-type semiconductor, the electrons diffuse from the $n$ layer to the $P$ layer. This leaves the positively charged atomic cores unbalanced on the $N$ side. Similarly, holes diffuse in the opposite direction creating a negatively charged region on the $P$ side in the vicinity of the junction. This, depleted of carriers region close to the junction is called space-charge region or, simply, depletion region.

Because the space charges on the two sides of the junction have opposite signs, there exists an electric field across the depletion region. If, for some reason, a free carrier appears in the depletion region it is accelerated by this electric field in the direction opposite to the diffusion currents. Currents of this type are called drifts.

When the junction of a photodiode is exposed to light, electron-hole pairs are generated that contribute to the drift current. Thus, illuminating the depletion region results in electrons being swept through the depletion region towards the negative electrode on the $n$ layer and similarly the holes gather at the positive electrode ($P$ layer). If the terminals of the photodiode are not open these two photocurrents add to produce a negative drift current commonly referred to as a photocurrent. Figure 5.2 shows the structure of a $P$ – $N$ photodiode. The depletion region is outlined and the arrows show the motion of the photo-generated carriers through the junction. In Fig. 5.3 a cross section through the junction is shown and the valence and conduction bands are outlined with the photo-generated carriers moving in the two bands, holes in the former and electrons in the latter.

It is possible to improve the performance of a photodiode by including a layer of intrinsic (not doped) semiconductor between the $N$ and $P$ layers. This increases the width of the depletion region which together with applied negative polariz-
Figure 5.2: Cross section of a semiconductor photodiode [89].

Figure 5.3: The P-N junction of a semiconductor photodiode [89].
tion voltage lead to a faster drift of the carriers through the junction leading to a better time response. The response is further enhanced by the resulting lower capacitance of the junction. This and other features of PIN photodiodes are listed in the subsequent sections where particular characteristics are described.

5.2.1 Equivalent circuit

![Photodiode equivalent circuit](image)

Figure 5.4: Photodiode equivalent circuit [89].

In Fig. 5.4 an equivalent electric circuit of a photodiode is shown. $I_L$ is a current source representing the photogenerated current, $I_D$ is the (typically reverse) current of the $P-N$ junction, $C_j$ represents the junction capacitance, $R_{sh}$ is shunt resistance and $R_s$ is series resistance of the photodiode. $C_j$ can be as low as several pF, $R_{sh}$ is $\sim 10^7$ to $10^{11}$ $\Omega$ and $R_s$ is several ohms.

The load current, $I_0$, can be calculated from

\[
I_0 = I_L - I_D - I' = I_L - I_s \left( \exp \left( \frac{eV_D}{k_BT} \right) - 1 \right) - I',
\]  

(5.2.1)

where $I'$ is the the shunt resistance current, $V_D$ is the junction voltage and $V_0$ is the voltage measured on the terminals of the device which is $V_D$ minus the drop on the series resistance. The term in braces stems from the thermal dependence of the junction current, $I_s$ being the photodiode reverse saturation current and $e$, the electron charge.

Open circuit voltage

Setting $I_0 = 0$ in Eq. 5.2.1 allows us to calculate the open circuit voltage

\[
V_{oc} = \frac{k_BT}{e} \ln \left( \frac{I_L - I'}{I_s} + 1 \right).
\]  

(5.2.2)
The logarithmic dependence on the photocurrent and the dependence on temperature make it easy to understand why the photodiode is typically not used in a voltage source configuration.

**Short circuit current**

Similarly, putting $R_L = 0$ in Eq. 5.2.1 yields the short circuit current

$$I_{sc} = I_L - I_S \left( \exp \frac{e I_{sc} R_S}{k_B T} - 1 \right) - \frac{I_{sc} R_S}{R_{sh}}, \quad (5.2.3)$$

where we substituted $V_D = I_{sc} R_S$. At room temperature, bearing in mind the typical values of $I_{sc}$ and $R_S$ we can drop the last two terms. The short circuit current can thus be very well approximated by the photocurrent itself.

### 5.2.2 V-I characteristics

![Photodiode V-I curves as the illumination is varied](image)

**Figure 5.5**: Photodiode $V$-$I$ curves as the illumination is varied [89].

Figure 5.5 shows the photodiode current as a function of the voltage for several values of light intensity. The open circuit voltage and the short circuit current are marked. The nonlinear dependence of $V_{oc}$ can be easily observed.
5.2.3 Photocurrent versus light intensity

In Fig. 5.6 the short circuit current is plotted as a function of the incident amount of light. This dependence is extremely linear. In fact it can span as much as 9 orders of magnitude, a range of values not found easily in other physical processes. The main reason for this great linearity is that the generation of electron-hole pairs is a one to one quantum process. More precisely, a single photon may generate a single carrier pair or not generate any but for a given diode the probabilities of these events are fixed.

The lower limit of linearity is given by the reverse current of the junction and can be inferred from the noise equivalent power, a key parameter of every photodetector defined below. On the high end the maximum optical power is given by

$$P_{\text{sat}} = \frac{V_{Bi} + V_R}{(R_s + R_L)S'}$$

(5.2.4)

where $V_{Bi}$ is the contact voltage ($0.2 – 0.3$ V), $V_R$ is the reverse voltage and $S$ is the (wavelength-dependent) photosensitivity in units of A/W. This simple expression is difficult to encounter in most texts on photodetectors. The saturation power depends on the given photodiode and is typically in the range of several mW.
The linearity can deteriorate if the intensity is too high at some point of the device as in the case of a tightly focused beam. In this situation the material saturates locally causing the upper linearity limit to decrease.

5.2.4 Dark current and shunt resistance

In the absence of illumination and for external polarizing voltage equal to zero (short circuit conditions) the net current through the junction must be zero. However, as soon as a finite polarizing voltage is applied a small current starts flowing. Its value depends on a given device and, obviously, the smaller the better. For practical reasons this *dark current*, $I_D$ is measured at -10 mV. This is the value encountered in data sheets. Alternatively, a shunt resistance can be specified, $R_{sh} = \frac{10 \text{ mV}}{I_D}$. As V-I characteristics of a $P - N$ junction are not linear thus determined value of shunt resistance is only valid around the zero voltage as depicted in Fig. 5.7.

**Figure 5.7:** Photodiode V-I curve at zero illumination and for small $V$. Both dark current and shunt resistance are specified at 10 mV [89].
5.2.5 Noise properties

The fundamental source of noise in a photodiode arises because of the corpuscular character of the carriers. It is the shot noise. Shot noise of a constant current is given by the general formula

\[ i_s = \sqrt{2eIB}. \]  

(5.2.5)

We have assumed here that the charge of the carriers is equal to the electron charge, \( e \), which is correct for both types of carriers in semiconductors. The presence of the elementary charge in this expression indicates the fundamental source of the shot noise. The bandwidth, \( B \), appears in Eq. 5.2.5 because the power spectrum of the shot noise is uniform, i.e. shot noise is a white noise.

The second form of noise encountered in the photodiode is the thermal or Johnson noise. Johnson noise is always present if there are resistive elements in the circuit. In the case of a photodiode, it is due to the shunt resistance

\[ i_j = \sqrt{\frac{4k_BT B}{R_{\text{sh}}}}. \]  

(5.2.6)

Thermal noise is also a white noise. Because of this its unit, likewise the unit of shot noise, is \( \text{A}/\sqrt{\text{Hz}} \).

The two types of noise are statistically independent and as such add in square

\[ i_n = \sqrt{i_{sD}^2 + i_{sL}^2 + i_j^2}, \]  

(5.2.7)

where the shot noise has been split into the shot noise of the dark current \( i_{sD} \) and the shot noise of the photocurrent \( i_{sL} \), again both independent. While the last two terms in Eq. 5.2.7 can be minimized by improving the construction of the device, the first term will always remain. A detector for which \( i_{sD}^2 + i_j^2 \leq i_{sL}^2 \) will be named "shot-noise-limited".

**Noise Equivalent Power (NEP)**

Noise equivalent power is defined as the optical power required to generate a photocurrent equal to the noise current as expressed in Eq. 5.2.7 with \( i_{sL} \) set to 0.

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1 In most literature the shot noise and thermal noise are presented as completely different phenomena without any connection between the two. An illuminating description proving the opposite can be found in [90].
Using the photosensitivity, \( S \), NEP can be written down as\(^2\)

\[
\text{NEP} = \frac{i_n}{S}. \quad (5.2.8)
\]

The unit of NEP is \( \text{W} / \sqrt{\text{Hz}} \). Good, small photodiodes, can have NEP as low as \( \sim 10^{-15} \text{ W} / \sqrt{\text{Hz}} \).\(^3\)

Because, according to the definition, the shunt resistance is inversely proportional to the dark current, it follows from Eq. 5.2.6 that \( i_j \propto \sqrt{i_D} \). But the shot noise of the dark current scales exactly in the same way (see Eq. 5.2.5). Therefore \( \text{NEP} \propto \sqrt{i_D} \) or, alternatively, \( \text{NEP} \propto 1 / \sqrt{R_{sh}} \). Figure 5.8 shows this dependence for a typical photodiode.

![Figure 5.8: Noise equivalent power (NEP) as a function of shunt resistance, \( R_{sh} \). [89].](image)

If a particularly low NEP is required, an appropriate choice is a PIN photodiode. In PIN photodiodes the dark current, and consequently NEP, are reduced owing to the high resistance of the \( I \)-type layer.

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\(^2\)It is in fact not necessary to stress the condition \( i_{sL} = 0 \), because for \( I_L = i^2_{sD} + i^2_f \) we have \( i_s L^2 = 2eI_B \ll i^2_{sD} + i^2_f \).

\(^3\)For example Hamamatsu S5971-3 series.
5.2.6 Speed

There are three time constants limiting the time response of a photodiode. Each is associated with a separate physical mechanism. Put together they determine the observed rise time which is important for digital applications or the cut-off frequency.

**Terminal capacitance and load resistance, $t_1$**

Terminal capacitance, $C_t$ is a parameter quoted in photodiode specifications. It is a combined capacitance of the photodiode package and that of its junction. By examining the equivalent circuit, Fig. 5.4 it is easy to see that $C_t$ together with the load resistance $R_L$ form a low pass filter\(^4\). The cut-off frequency of this filter is $f_c = \frac{1}{2\pi R_L C_t}$. Equivalently, the rise time defined using the typical definition used in digital electronics of 10% to 90% change in the output signal is given by

$$t_1 \approx 2.2 R_L C_t. \quad (5.2.9)$$

In order to minimize the product $R_L C_t$ a circuit with a small load resistance (see Sec. 5.3) can be used and a diode with small $C_t$ should be selected. The junction capacitance, $C_j$, is given by purely geometrical factors. It is proportional to the active area of the diode and it decreases with increasing the width of the depletion region. In turn, the width of the depletion region is proportional to the product of the resistivity and applied reverse voltage. Thus a small photodiode with a reasonably large reverse bias is the correct choice for high speed applications. The small size will also prevent excessive noise by reducing the dark current. On the other hand large reverse voltage increases the dark current which has a detrimental effect on the noise performance of the device. Again the high resistance of the $I$-type layer in a PIN photodiode will help keep both the dark current and the junction capacitance low.

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\(^4\)It often happens that beginners are not aware of this simple fact. In an attempt to increase the output voltage they increase $R_L$ which necessarily reduces the bandwidth. A common example is when a photodiode at the end of a Fabry-Perot spectrum analyzer is terminated with too high resistance. The observed transmission peaks are then much broader than expected from the finesse of the cavity and also asymmetric. This is often, incorrectly, taken as an effect of cavity misalignment.
**Diffusion time of carriers generated outside the depletion region, \( t_2 \)**

If the incident light illuminates regions outside the active area, carrier pairs are generated not only in the depletion region but also in the non-depleted portion of the semiconductor. These carriers will also contribute to the photocurrent, however because they have to diffuse to the depletion region, they will be significantly slower. Hence, the value of this diffusion time, \( t_2 \), is not simply a parameter of the photodiode but rather depends on the particular illumination used. In some cases it may easily reach several microseconds.

The "slow carriers" generated outside the depletion region can be very easily observed if a balanced detection scheme is used. Despite careful balancing the output of the device may show a "long tail" response if the diffusion time, \( t_2 \), of the two photodiodes is different, e.g. due to a slight misalignment of one of them. The cause of the slow response can be corroborated by taking the integral over a light pulse. The difference in the \( t_2 \) of the two photodiodes results in a non-zero output signal which may be longer than the pulse itself. However, if the powers are balanced, the integral over the output signal remains zero. These effects have been regularly observed if the balanced detector described in subsequent sections was connected to a pair of relatively large Hamamatsu S3883 photodiodes.

**Transit time through the depletion region, \( t_3 \)**

Finally, the time that a carrier takes to cross the depletion region is finite and depends on both the width of this region and the electric field therein. In turn, the last two depend on the applied reverse voltage. This transit time is denoted by \( t_3 \).

The three time constants combine to produce the effective rise time

\[
T_r = \sqrt{t_1^2 + t_2^2 + t_3^2}.
\]

(5.2.10)

From this the useful cut-off frequency, \( f_c \), can be determined. For fast photodiodes, like Hamamatsu S5973, \( f_c \) may be in the range of gigahertz.
5.2.7 Two regimes of operation

In normal operation the photocurrent is a function of the incident amount of light only and is effectively independent of the applied voltage\(^5\). Therefore an external bias voltage is not absolutely necessary. A photodiode operated with zero bias acts like a solar cell and this regime is called photovoltaic. Nevertheless, negative bias voltage is usually applied in order to improve the speed (and also the linearity) as described earlier. However, this unavoidably increases the dark current whose shot noise deteriorates performance of the device in low noise applications. Therefore the photovoltaic regime should be considered when low noise is the fundamental requirement.

5.3 Photodetection electronics

The simplest photodetector circuit consists of a photodiode and a load resistor, \( R_L \). The diode, with reverse bias or not produces a photocurrent, \( I_0 = SP \), where \( P \) is the optical power. This current produces a voltage drop, \( V_0 = I_0 R_L \) which can be further amplified (Fig. 5.10(a)). As explained in Sec. 5.2.6 the bandwidth of this simple circuit is limited by the low-pass filter formed by parallel connection of the terminal capacitance of the diode, \( C_t \), with the load resistance.

It is therefore desirable to reduce \( R_L \) in order to maintain the maximum avail-

\(^5\) Sometimes beginners find it surprising when a passive photodetector runs despite the batteries being removed.
able speed. Unfortunately, reducing $R_L$ decreases the output voltage. A way out of this situation is to use a transimpedance amplifier. The photocurrent is sent directly to the (virtual) ground of the inverting input of an operational amplifier (Fig. 5.10(b)). Because the input does not draw any current, the output of the amplifier is forced to produce across the feedback resistor a voltage $V_f = -I_{sc}R_f$, where the photocurrent is denoted by $I_{sc}$ to emphasize that the photodiode works in a (virtual) short circuit configuration.

The transimpedance is perhaps the most commonly encountered configuration. Although the basic idea is fairly simple, the circuit often tends to self-oscillate and eliminating this unwanted behavior often means a significant sacrifice of the bandwidth.

5.3.1 Inspiration — CSA with a pulse shaping stage

The charge sensitive amplifier (CSA) has been invented for applications in particle physics and as such it is usually used with solid-state particle detectors rather than photodetectors. However, it is possible to adapt the circuit so that it can be used with photodiodes. From the electrical point of view the main difference between a particle solid state detector and a photodiode is that the capacitance of the former is a few orders of magnitude larger.

There has been a lot work done on the CSA signal processing in the context of nuclear physics (see e. g. [91], [92] and dedicated devices including the following pulse shaping stages can be purchased from various manufacturers$^6$. In nuclear physics the CSA is used to detect and measure particles’ energy. This energy is absorbed in a (solid-state) detector which produces an electric pulse.

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$^6$Amptek, Cremat, Hamamatsu, to list a few.
This pulse is usually very short (a fraction of a ns). The integral of this current pulse, or the charge, is proportional to the particle’s energy. Therefore it is this charge that the nuclear scientist has to measure and this is the task of the CSA. In fact the circuit of the CSA is just that of a current integrator.

### 5.3.2 Electronic design

![Schematic of a charge-sensitive amplifier.](image)

Figure 5.11: Schematic of a charge-sensitive amplifier.

\[ v_{\text{OUT}}(t) = -\frac{1}{C} \int_0^t i_{\text{IN}}(t') dt' \]

Once the charge has been integrated the output voltage of the CSA or more precisely the amplitude of the output voltage jump, has to be determined. The idea is to create a smooth (typically an approximate Gaussian) and reasonably long (about 1 µs and always fixed) pulse whose amplitude is easy to measure. This amplitude is proportional to the CSA voltage jump and thus the absorbed particle’s energy. Making the pulse duration constant and the shape smooth ensures no errors in the measurement of the amplitude. This pulse is produced in a pulse shaping stage that follows the CSA. Pulse shaper consists of a number of differentiating and integrating stages with carefully adjusted time constants including a pole-zero stage just after the CSA.

Left alone the CSA would saturate after some number of detected particles and the device would become useless. Therefore its output has to be reset. It is to say the integrating capacitor has to be discharged. There are two methods of doing this. Passive and active.

The passive method consists in connecting a resistor in parallel with the capacitor. The resistance has to be chosen such that the time constant of the resulting RC circuit is much longer than it takes the output of the CSA to jump. On the
other hand this time constant must be short enough so that it effectively protects the CSA output from saturation. The range of typical values is from a couple of µs to a few hundred µs.

Active reset can be done electronically or optically. We do not go into details as they are not of our interest here, however it is worthwhile mentioning the so called baseline restoration circuit which brings the output of the CSA to the initial state in a feedback loop. This is desired from the point of view of particle detection as particle arrival times are random. Therefore there is always a trade off in the precision of the energy measurement and the ability of the circuit to detect two particles coming in a short time interval one after the other.

The use of the CSA is not limited to particle physics. Apart from measuring gamma radiation and soft x-rays, it has been successfully applied in the domain of quantum optics. Hansen et al have built a balanced homodyne detector for application in ps-pulse measurements [93] based on a commercial CSA and a pulse shaping stage from AMPTEK. The same CSA plus pulse shaper design has also been used in µs-domain but this gives rise to several complications [94, 95].

### 5.3.3 Noise considerations

In this section we only briefly describe the general approach to the characterization of noise. A more detailed analysis is deferred to section 5.4.6 where we describe the noise performance of the constructed circuit.

**Noise per unit bandwidth**

The resistance load and the transimpedance circuits introduced earlier produce at the output a voltage waveform that has the same time dependence as the light intensity. Noise in this kind of continuous-time circuitry is usually defined in units of V/$\sqrt{Hz}$. This comes from the fact that both Johnson (thermal) and the electron shot noise are forms of white noise. In other words the amount of (electrical) power per unit bandwidth is constant. Since electrical power is proportional to the square of the voltage, and voltage is what one usually measures, the commonly accepted unit is V/$\sqrt{Hz}$ rather than W/Hz.

Thus far about electronics. When it comes to electro-optics one encounters a
unit of $W/\sqrt{Hz}$ which at first appears to be wrong. It is not wrong, though. It comes from the fact that photodiodes produce current proportional to the optical power. This current is then transformed into voltage. This way noise measured at the output in $V/\sqrt{Hz}$ gets transformed into $A/\sqrt{Hz}$ and this one into $W/\sqrt{Hz}$, where $[W]$ stands for a Watt of optical power. This way we arrived at the definition of NEP in Sec. 5.2.5.

**Noise per unit time**

There exists another way to characterize the noise of a device. If we integrate noise that has a flat power spectrum during some fixed time interval $T$, the variance of the integrated value will grow linearly with $\sqrt{T}$. Equivalently, the standard deviation will grow proportionally with $T$. Thus noise measured in this way can be given in $V/\sqrt{s}$.

**Noise per event**

A different description is required if the measured quantity appears in short bursts like in the case of the measurement of energy of nuclear particles. The commonly used in such situations CSA produces a charge that is proportional to the energy’s particle. Therefore it is natural to characterize the uncertainty (or noise) of this measurement in charge units per event. This is why CSA’s noise is given in rms electrons. Knowing the sensitivity of the detector used this can be transformed into keV, which is the most useful form for a nuclear physicist.

### 5.4 Photodetector

Our aim is similar to that of the nuclear physicist trying to measure the energy of a particle. Both the QND and the macroscopic polarization measurements produce optical pulses that require integration. The fundamental difference is that these pulses are, at least from the perspective of electronics, quite long, on the order of a $\mu s$. Therefore a naive application of a tandem CSA plus pulse-shaping designed for particle detection may not be an optimal solution. For example, a pole-zero cancellation circuitry typically employed in these systems are difficult
to tune and if this is not done well significant errors might be introduced. There is also another factor that makes the use of the shaping stage in a pulse photodetector obsolete. While detection of a nuclear particle is a random event, pulses in our experiments come only when we send them. This, combined with a fast data acquisition system with memory, makes it possible to record the output of the CSA only when its state changes and extract the amplitudes of the voltage jumps in a data analysis process. With reasonably fast data processing thus can be done even in real-time.

### 5.4.1 Electronic design

![Simplified diagram of the photodetector](image)

**Figure 5.12:** Simplified diagram of the photodetector (see text).

Figure 5.12 shows the basic design of the photodetector. The currents produced by photodiodes PD1 and PD2 are subtracted and fed into the CSA through the coupling capacitor $C_c$. Since $Q_c / Q_t = C_c / C_t$, in order not to lose signal one demands $C_c \gg C_t$.

The CSA consists of an operational amplifier U1 together with a capacitor C1 and a resistor R1. R1 slowly discharges C1 thus preventing saturation of the operational amplifier. The output signal of the CSA requires further amplification. This is done with another operational amplifier, U2, that is configured as a non-inverting voltage amplifier. Its gain is given by $R_3 / R_4 + 1$. Output of the CSA is coupled to the voltage amplifier stage through a high-pass filter built with R2.
and C2. For fast signals the two photodiodes can be modeled as in 5.13(b). Here,

**Figure 5.13:** Two equivalent circuits of the parallel connection of two photodiodes.

only the shunt resistance \( R_{\text{sh}} \) and the terminal capacitance \( C_t \) are included. The series resistance of both photodiodes is omitted as its influence is negligible. In fact, a basic understanding of the circuit operation is obtained even when the two photodiodes are approximated by just a current source 5.13(c). This is a consequence of the very value of the shunt resistance (1 GOhm) and a very small terminal capacitance (several pF). We use this model for the analysis in the next section.

The very large value \( R_{\text{sh}} \) implies that charge collected on \( C_t \) does not decay significantly when compared to other time constants in the circuit. The actual values of \( R_{\text{sh}} \) and \( C_t \) result in a time constant of about a few milliseconds. The values of \( R_{\text{sh}} \) and \( C_t \) are respectively half and double of the corresponding values for a single photodiode. This is because the photodiodes are effectively connected in parallel. The actual series resistance and terminal capacitance of a single photodiode at zero bias have been extrapolated from the curves given in the data sheet of HAMAMATSU S5973.

The model in 5.13(b) is used later in a PSPICE simulation (Sec. 5.4.5) to account for some characteristic features present in the measured signal that cannot be explained by the analytical model presented in the next section.
5.4.2 Circuit analysis

The circuit in 5.12 is sufficiently simple to be solved analytically. We model the photodiodes as in 5.13(c). Consequently the coupling capacitor, $C_c$, is also omitted.

Considering the part of the circuit that includes the input current source and the CSA constructed with $C_1$, $R_1$ and $U_1$, it is easy to show that the output voltage of this stage is given by

$$v_1(t) = v_1(t = 0) - \frac{1}{C_1} \int_0^t i_1(t')dt',$$  \hspace{1cm} (5.4.1)

with the current of the capacitor equal to

$$i_1(t) = e^{-\frac{t}{R_1C_1}} (i_L(t' = 0) + \int_0^t \frac{di_L}{dt'} e^{\frac{t'}{R_1C_1}} dt').$$  \hspace{1cm} (5.4.2)

It is clear that for times $t \ll R_1C_1$, the output voltage is given by the integral of the input current. This is the basic function of a CSA. Electronically a CSA is nothing but a current integrator with the gain factor given by $-1/C_1$. Therefore $C_1$ is usually chosen as small possible. With the values given in 5.12, $C_1$ discharges with a time constant, $\tau_1 = 300 \mu$s. This time constant is adjusted with $R_1$ and must be set longer than the sum of the pulse duration and the time necessary for a reliable readout.

$C_2$ and $R_2$ form a high pass filter coupling the output of the CSA to the following voltage amplifier stage. The values are chosen such that $\tau_2 = R_2C_2 = 1$ ms $> \tau_1$. This ensures that pulses which are much shorter than $\tau_1$ pass through the filter without distortion and the time available for data acquisition is not restricted by the filter itself. The output voltage of the filter is given by

$$v_2(t) = -v_{C_2}(t = 0) + e^{-\frac{t}{R_2C_2}} (v_1(t' = 0) + \int_0^t \frac{dv_1}{dt'} e^{\frac{t'}{R_2C_2}} dt').$$  \hspace{1cm} (5.4.3)

For times $t \ll R_2C_2$, $v_2(t)$ is just equal to $v_1(t)$ (assuming $C_2$ is initially discharged).

Figure 5.14 shows the response of the circuit to a single square and 2 $\mu$s-long current pulse. The traces are calculated using the above analytical expressions. The same signal is shown at three different time scales. In part (a) we see a perfect integration function. Observing the signal for a longer time shows the 300 $\mu$s
Figure 5.14: Calculated response of the photodetector circuit to a single 2 μs-long square pulse and an imbalance on the photodiodes resulting in a 1 nA input current difference. (a), (b), (c) show the same set of traces at various time scales.
CHAPTER 5: DETECTION

Figure 5.15: Calculated response of the photodetector circuit to a series of 20
2 µs-long pulses at 20 µs duty cycle. (a) and (b) show the same set
of traces at various time scales.

discharge (b). Finally, in (c) the time scale is long enough to see the influence of
the high pass filter.

In Fig. 5.15 a series of 20 2-µs pulses with a duty cycle of 20 µs is used. As new
pulses arrive the output reflects the integral over the entire signal. However, the
CSA capacitor, $C_1$ discharges continuously and this discharge is visible between
consecutive pulses. This happens despite the fact that the duty cycle is much
shorter than the discharge time constant. However, for the same reason, the
exponential discharge can be approximated by a simple linear function of time.
This allows for the discharge to be removed in a simple data processing routine
(Sec. 8.1 in Ch. 8). Note that after a long time, the output waveform is essentially
the same as for a single time.

Figure 5.16: Calculated response of the photodetector circuit to a series of 200
2 µs-long pulses at 20 µs duty cycle. (a) and (b) show the same set
of traces at various time scales.
Figure 5.16 goes to the regime of very long pulse trains. A train of 200 pulses with otherwise the same characteristics as before is used. This results in a steady state at the output of the CSA. Every pulse contributes exactly the same amount of charge as is lost during one duty cycle. Note that the output of the high pass filter exhibits a different behavior in that the DC component is gradually removed.

5.4.3 Implementation

There are two main criteria for a photodetector used in pulsed QND measurements. First, the electronic noise must be lower than the photon shot noise of the light pulses. This is also important if the detector is to be used also for detection of macroscopic polarization rotation signals (Ch. 8). In this case the lower the electronic noise the better as this ensures good resolution even at very low optical power, which may be desirable. The other criterion is the dynamic range. The higher the upper limit for the optical power and pulse duration (or equivalently the photon number) that can be handled without saturation the better as this gives the user flexibility in respect to these parameters.

We have decided to use a commercial CSA owing to its superb noise characteristics. AMPTEK A250 is a hybrid CSA chip which, if other elements of the circuit are appropriately chosen, can provide noise as low as $\sim 10^2$ electrons per pulse ([96], App. B). The first stage of A250 works with an external FET transistor. The choice of the transistor is crucial as it determines the noise performance of the chip. In general best results are obtained with FETs that have low capacitance. However, this capacitance must be matched to that of the sensor, in our case the photodiodes ([96], App. B).

We have tested the performance of the CSA with several of the transistors recommended by AMPTEK and also with a few different photodiodes. Best results have been obtained with FET 2SK152 from SONY and PIN photodiodes S5973 from HAMAMATSU. Both elements are characterized by very low capacitance $\sim 1$ pF. The dark current of S5973 is typically $\sim 1$ pA. This remarkably low value renders the dark current noise negligible. For instance, in a typical pulse of 1 $\mu$s a 1 pA dark current will transfer $10^{-18}$ C, that is about 6 electrons. The error in the charge integrated by the CSA due to the shot noise of the dark current is given by square root of the transferred charge. Even if we take into account that
the noise of the two photodiodes adds in quadrature which results in a value larger by a factor $\sqrt{2}$ and that the pulses may be several $\mu$s long, the dark current contribution to the noise of the device remains smaller than 10 electrons per pulse.

In fact, S5973, are fast enough to be used in photovoltaic regime (Sec. 5.2.7) where the dark current contribution to the electronic noise can, at least in principle, be completely eliminated. The difficulty in working with these photodiodes is their very small active area which requires very good focusing of the beams. As an alternative a bigger photodiode can be used. This is sometimes very handy. For example, the polarization rotation signal caused by atoms can be used as a diagnostic for fine tuning of the probe alignment to the atomic sample. With the small S5973 photodiodes this is completely impractical as the probe immediately goes off their active area.

For these purposes we sometimes used a large active area HAMAMATSU S3883. These are among the best photodiodes available on the market given their large active area. However, the larger size inevitably comes at the cost of larger dark current. For S3883 this is typically about 50 times higher than for S5973. This could be remedied if the diodes could be used in photovoltaic regime. Unfortunately, it turns out to be impossible. The balanced configuration of the detector makes it very sensitive to differences in speed of the two photodiodes whose currents are subtracted. In particular it is our experience that the diode receiving more light always tends to contribute more “slow” carriers that have to find their way to the depletion region by diffusion. Unless the separation between consecutive pulses is made very long, this leads to significant errors.

Figure 5.17 shows the electronic schematic of the detector with the actually used components. The voltage amplifier stage is built with an operational amplifier A275 from AMPTEK. This chip has been chosen simply because it was purchased for an earlier version of the detector circuit and was no longer used. Otherwise a cheaper replacement could be used.

### 5.4.4 Parameters

In this section the principal characteristics of the circuit are listed. These have been computed given the circuit parameters. An important parameter is the gain.
Figure 5.17: Schematic of the detector circuit with actual elements.

Its calculated value is compared to the measured one in Sec. 5.4.5.

**Photodiode sensitivity**

The photo-sensitivity of S5973 photodiodes at 780 nm is 0.51 A/W which corresponds to quantum efficiency of 0.81.

**CSA gain**

This is given solely by the capacitance of the CSA capacitor, C1, and equal to \(1/C_1 = 10^{12} \text{ V/C}\) or 1 \(\text{mV/µs nA}\).

**Voltage amplifier gain**

\(G = 1 + R_3/R_4 = 11\).

**Total photo-sensitivity**

Collecting the above values yields 5.5 \(\text{mV/µs nW}\) or 1.40 \(\mu\text{V/photons}\) at 780 nm.
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Time constants

The CSA discharge time constant is $R_1C_1 = 300 \, \mu s$. The high pass filter time-constant is $R_2C_2 = 1 \, ms$.

5.4.5 Performance

In this section the response of the photodetector described earlier is measured and the results are compared to the calculations in Sec. 5.4.2. All the data presented here have been acquired with the detector slightly unbalanced. A total power of a few $\mu W$ has been split between the two photodiodes and the imbalance has been adjusted arbitrarily.
Figure 5.18: Measured response of the photodetector to a single 1 μs-long square pulse. (a) negative imbalance, (b) positive imbalance on the photodiodes. (c) and (d) show the same sets of traces as respectively (a) and (b) at a longer time scale. Red trace is the total optical power.
In Fig. 5.18 the detector is exposed to a single optical pulse of 1 µs duration. Plotted are the output voltage (green) and the voltage measured directly at the output of the CSA (blue). A good agreement with the calculated traces in Fig. 5.14 is observed. The response of the circuit to a positive and negative imbalance is shown. This demonstrates that the circuit behaves in the same way irrespective of the polarity\(^7\). A particular feature of the A250 chip is that it exhibits a constant offset at the output, about -520 mV. This value does not fluctuate and therefore does not affect the operation of the circuit.

In Fig. 5.19 the photodetector receives a series of 20 pulses of the same length as before, one every 20 µs. Again by comparison to Fig. 5.15 a good agreement with calculations is observed.

Figure 5.20(a) shows the response of the device when the input consists of an infinite series of pulses of the same characteristics as before. In this case a clear difference between the predicted (Fig. 5.16) and the observed behavior is observed. According to the model the output voltage of the CSA should reach a steady state in which the decay due to the discharge of the CSA capacitor exactly balances the voltage jump due to every pulse. This indeed happens, however, unlike in the calculated traces where the DC component levels off at a non-zero value, in Fig. 5.20(a) it is removed during the pulse series. Consequently, the DC component at the output of the detector (i.e. after the voltage amplifier) instead of decaying to zero with a time constant given by the high pass filter coupling the CSA to the voltage amplifier stage, exhibits a bump.

This effect only occurs for signals of definite polarity and becomes visible at long time scales. Because of this its influence on the overall performance of the detector can be removed by the same technique that is used to remove the effects of the other slow dynamics in the circuit (Sec. 8.1 of Ch. 8).

Although the unexpected decay of the DC component at the output of the CSA can be easily accounted for and therefore does not lead to errors, it remains interesting to find the cause of the effect. Fig. 5.20(b) shows the same effect when bigger and thus more capacitive, HAMAMATSU S3883, photodiodes have been used. As can be observed, although the particular shape of the trace is slightly different, the overall behavior is the same. The DC offset at the output of the photodiode is approximately -520 mV.

\(^7\)However, as the imbalance is not calibrated, the amplitudes of the signals should not be directly compared. Photo-sensitivity measurements are reported in the following section.
Figure 5.19: Measured response of the photodetector to a series of 20 1 μs pulses at 20 μs duty cycle. (a) and (b) show the same set of traces at various time scales. Red trace is the total optical power.
Figure 5.20: Measured response of the photodetector to an infinite series of 1-µs pulses at 20 µs duty cycle. (a) HAMAMATSU S5973, (b) HAMAMATSU S3883.
Figure 5.21: Simulated response of the photodetector to an infinite series of 1-μs pulses at 20 μs duty cycle for $R_{sh}$ set to an unrealistically low value (see text).

CSA is removed and the detector output shows a bump\textsuperscript{8}. It has been found that a similar behavior can be obtained by simulating the circuit using a more complete equivalent circuit of the photodiodes than that used in the analytical analysis.

The model in Sec. 5.4.2 was based on the oversimplified equivalent circuit, Fig. 5.13(c). In Fig. 5.21 the result of a PSPICE simulation of the circuit using a more complete equivalent circuit 5.13(b) is shown. To obtain results similar to the observed in Fig. 5.20 the shunt resistance of each of the photodiodes has been modeled by an unrealistically low 2 MΩ. This simulation suggests that the loss of the DC component at the output of the CSA is caused by a “leak” of charge through the photodiodes. The charge collected on the coupling capacitor $C_C$ is lost and because the charge on the CSA capacitor must be the same it also decays.

A realistic value of the shunt resistance is rather in the range of GΩ and the explanation above can only be treated as a hint. On the other hand, it does suggest a more probable scenario. Note that the last model still does not contain the most complete equivalent circuit of the photodiodes, Fig. 5.4. In fact, as the charge is collected on $C_C$ one of the photodiodes acquires a very small positive bias and as a result starts conducting. This little current discharges $C_C$ and this way causes the observed effects. This conjecture is further confirmed by Fig. 5.20(b). As

\textsuperscript{8}An exact quantitative comparison of the traces in Fig. 5.20(a) and in Fig. 5.20(b) is not possible as the imbalance was different in each case.
expected, the scale of the effect depends on the photodiodes used.

It can be inferred that a negative bias would be helpful in this situation. However, as the observed effects merely affect the long time dynamics they are automatically removed in the process of data analysis (Sec. 8.1 of Ch. 8). This is to be contrasted with solutions employing electronic pulse shaping. In this case every change in the sensor would require corresponding adjustments in the electronics.

**Photo-sensitivity measurements**

In section 5.4.4 the expected total photo-sensitivity has been calculated. This value directly depends upon the CSA capacitor, $C_1$, and because this in our design is only 1 pF, it would not be a surprise if the actual photo-sensitivity of the detector were lower than calculated. In fact, soldering pads on a PCB can easily have a capacitance larger than 1 pF. This stray capacitance combines with $C_1$ which results in a lower gain.

The photo-sensitivity of each channel has been measured independently by focusing a single beam on one of the photodiodes at a time while the other has been kept covered. The power in the beam has been measured by a powermeter. This has given the following results.

$S_+ = 2.1 \pm 0.3 \text{ mV} \mu \text{s nW}$ or $0.53 \pm 0.08 \text{ µV photon}$ at 780 nm. $S_- = 2.5 \pm 0.6 \text{ mV} \mu \text{s nW}$ or $0.63 \pm 0.17 \text{ µV photon}$ at 780 nm.

As expected, the measured values are smaller than the calculated one. The relatively large uncertainties reflect the fact that absolute measurements at nW levels of power are difficult. In the remainder of this thesis the weighted value of the photo-sensitivity is used. $S = 2.2 \pm 0.3 \text{ mV} \mu \text{s nW}$ or $0.55 \pm 0.07 \text{ µV photon}$ at 780 nm.

**5.4.6 Noise performance**

**Frequency domain analysis**

The easiest way to measure the noise produced by a device is done with a spectrum analyzer. The spectrum analyzer gives a power spectrum of the measured
signal, in this case a noisy signal. Assuming that the device is perfectly linear, there will be no intermodulation of the signal and the noise and so one could, in principle, calculate the noise in the time domain for a pulse of any given duration, $T$, based on the measured power spectrum. This is especially easy if the noise has a flat spectrum (white noise). Integrating the output over time $T$ will then give a value proportional to the square root of $T$ (see e.g. [97]).

In practice no noise is really white noise. Two well known types of noise which are thermal noise and shot noise have a flat spectrum, however the circuit always imposes some frequency limitations. Although always present, these bandwidth limiting factors are not what makes the transition from the frequency domain to the time domain difficult. Not all types of noise have a flat spectrum. The very common in electronics, so called pink or flicker, noise has a power spectrum that scales like $1/f$. One important source of flicker noise is the carrier generation-recombination process in semiconductors [98]. What ultimately invalidates the transition from the frequency domain to the domain is the loss of phase information. If one uses a standard scalar spectrum analyzer only the power spectrum is obtained which carries no phase information. This would be no harm if we could assume that all the frequency components are uncorrelated i.e. that they all add incoherently. However, this assumption may only be justified by phase-sensitive measurements which cannot be performed on a simple scalar spectrum analyzer. For the aforementioned reasons and because we work with pulses, the time domain analysis of the noise is preferred. Nevertheless, for the sake of completeness, we include the power spectrum measurements. As can be seen in Fig. 5.22, the spectrum is not completely flat and thus we are not dealing here with pure white noise. It also does not correspond to an integrated white noise. In a log-log plot a flat spectrum input noise upon integration would give a straight line with a slope of 20 dB/dec.

We now proceed to the time domain description and the per event description.

**Time domain and per event analysis**

The circuit under consideration is build upon a CSA chip A250 that was designed to be used with particle detectors and thus its noise is characterized per detection event. According to the curve given in the specifications (App. B, Fig. 1), when
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**Figure 5.22:** Electronic noise spectra in linear (a) and logarithmic (b) scale. The lower curve (green) is the spectrum analyzer noise floor. The upper curve (blue) in (b) is the detector noise with Hamamatsu S5973. The two upper curves in (a) are the detector noise with Hamamatsu S5973 (green) and Hamamatsu S3883 (red).

used with the JFET 2SK152 and a detector of impedance about 6 pF which is the total capacitance of two not biased S5973, the noise should be something between 100 and 200 rms *electrons* per detection event.

In our application the chip is used to integrate relatively long pulses and therefore the description of noise per event is not guaranteed to be adequate. To determine how the electronic noise should be characterized in this situation we measure its value versus the integration time. The integration time is just the time between the two samples that we subtract from each other in order to determine the integral. As we can see from Fig. 5.23, the output noise of the entire circuit is a steadily growing function of the integration time $T$ with a large positive offset. This offset is the high frequency contribution. In this case the sampling rate was 5 MSa/s and so the minimum $T$ was 0.2 $\mu$s. Thus in this context "high frequency" refers to all those components that change faster than 0.2 $\mu$s. The bandwidth of the oscilloscope used for measurement was set to 20 MHz. This defines the upper limit of the frequencies that contribute to the offset. Since the offset stays constant as $T$ is varied it can be attributed to the output of the circuit.

On the other hand the small growing component comes essentially from frequency components lower than the sampling rate. These can originate either from the output or they may just be an integrated white input noise. In the first case the curve should stop growing and settle at some value of $T$ where no more
correlations are present. This would then correspond to the lower frequency limit of the output noise. On the other hand the integrated input noise should give a straight line as explained earlier.

From the data at hand it is difficult to judge what causes the growing component. The curve does not saturate which would indicate an integrated input noise. Yet it may be that longer $T$ are required to see the saturation. Whatever the origin of the growing component we note two things. One, it appears to be no harm to treat it as if it came from the input. Two, it is small enough to be neglected in most applications. One would then take the maximum value $8.5 \times 10^{-7} \text{[V}^2\text{]}$ as the equivalent noise per event. This is correct for $T$ up to 20 $\mu$s. Since we do not employ pulses longer than this, it makes a satisfactory description of the electronic noise of the system.

For all the measurements of the detector and also later for measurements of polarization rotation signals, the detector has been connected to a digital oscilloscope, LeCroy WaveRunner 64Xi. We saw in Fig. 5.23 that for the sensitivity setting 2 mV/div, the noise of the oscilloscope is about 50 times lower than the electronic noise of the detector. Although not visible in the graph, the oscilloscope noise does not change as the integration time is increased. In most cases 2 mV/div will be too high a sensitivity and one will use values about 20 mV/div. To that end the system detector plus oscilloscope must be treated as one device.
In Fig. 5.24 we show how the noise contributed by the two components varies with the sensitivity setting of the oscilloscope. As the sensitivity is decreased (or mV/div increased) the oscilloscope noise grows and at the setting of 20 mV/div reaches a value close to that of the detector’s noise. It will be shown later that in practice this large value of the noise contributed by the oscilloscope at small sensitivity is much below the light shot noise, assuming the sensitivity is adjusted reasonably to the optical power used.

![Graph showing electronic noise versus oscilloscope sensitivity. Crosses (blue): the oscilloscope noise. Circles (green): total noise of the oscilloscope and the detector. Lines are drawn to guide the eye. Bandwidth 20 MHz, sampling rate 5 MSa/s.]

**Figure 5.24:** Electronic noise versus oscilloscope sensitivity. Crosses (blue): the oscilloscope noise. Circles (green): total noise of the oscilloscope and the detector. Lines are drawn to guide the eye. Bandwidth 20 MHz, sampling rate 5 MSa/s.

**Noise reduction by averaging and fast sampling**

The results presented so far have been obtained with a DC coupled oscilloscope whose bandwidth was set to 20 MHz and the sampling rate was 5 MSa/s. The integrated signal was then given by the difference between the values of two samples. One before the pulse and the other afterwards.\(^{11}\) It will often be the case, that the separation between consecutive pulses will be large enough as to sample the output more than once. The experimental uncertainty can then be reduced by averaging.

\(^{11}\)Since we are not trying to reconstruct the original signal from the samples, the Nyquist theorem does not impose a limit here.
In this section we study the dependence of the electronic noise upon the averaging time and the sampling rate. Theoretically, for white noise, one expects the variance to decrease like $1/T_A$, where $T_A$ is the averaging time. As we can see from the graphs in Fig. 5.25, the noise indeed drops very fast as $T_A$ is increased from 0.2 µs to 1 µs. For $T_A$ longer than 1 µs, the decrease is much slower.
Figure 5.25: Electronic noise as a function of the averaging time and the integration time. Sampling rate (a): 5 MSa/s, (b): 10 MSa/s, (c): 20 MSa/s, (d): 50 MSa/s. Oscilloscope sensitivity 2 mV/div, bandwidth 20 MHz.
Another feature evident from the plots is that it only makes sense to sample faster if the noise to be reduced contains components around the sampling frequency. In this particular case, it can be observed that sampling at 10 MSa/s is better than sampling at 5 MSa/s, likewise 20 MSa/s is better than 10 MSa/s. However, at 20 MSa/s this tendency breaks and sampling at 50 MSa/s does not reduce the variance. The reason for this is simple. The low-pass filter of the oscilloscope has a cut-off frequency of 20 MHz.

With the 20 MHz low-pass filter in place, the effect of averaging over 3 µs is roughly to reduce the variance of the electronic noise by a decade. Most of this effect is already obtained for 1 µs averaging time. For the same 20 MHz filter the effect of increasing the sampling rate from 5 MSa/s to 20 MSa/s, is much smaller and amounts to a reduction in variance by about 20–30%. It should be noticed that this requires 4 times more memory to store the data and thus may not always pay off.

Based on the presented analysis, it appears reasonable to use a 5 MSa/s, sampling rate along with a 20 MHz low-pass filter and 1 µs or longer averaging. The next section provides concrete examples for averaging time of 1 µs and 3 µs.

### 5.4.7 Application to light shot noise measurements

The data points in Fig. 5.26 are variances of the voltage jump produced by the circuit upon receiving a 2.2 µs balanced optical pulse. These voltage jumps are a direct measure of the integral over the pulse.

Every point in the plot has been computed from a train of 1000 pulses. The duty cycle has been set to 100 µs. Since this is a noise measurement whose mean value is zero, there is no build-up of the signal at the output and thus the contribution from the slow discharge is negligible during the integration time. Therefore to determine the value of the jump a simple subtraction of the sample before the pulse from a sample afterwards is sufficient.

To the experimental data marked in the plot we fit a second order polynomial using the least square method. This is done in linear coordinates and only later plotted in a log-log scale. Below we explain the meaning of each of the three terms in the fit.
Figure 5.26: Total (light + electronic) noise measured in the setup of Fig. 5.27. The heavy solid (red) line is a second-order polynomial fit. Dotted (black) line is the constant term, light solid (green) line is the first-order term, dash-dot (blue) line is the quadratic term. The various dash lines represent electronic noise floor (detector + oscilloscope) at the oscilloscope sensitivity 2, 5, 10, 20, 50, 100, 200 and 500 mV/div. The color of each marker corresponds to the oscilloscope sensitivity at which respective data point has been acquired. Variance in [mV$^2$] converted to [photon$^2$] (red vertical axis) using the measured photo-sensitivity $S = 0.55 \frac{\mu V}{\text{photon}}$ (Sec. 5.4.5).

Figure 5.27: Setup used for the measurements of this section.
The constant term models the electronic noise, and as can be seen it coincides very well with the measured value of the electronic noise of the detector at the highest sensitivity, 2 mV/div.

The square term has two contributions. Both require imperfections in balancing. First, a constant imbalance makes the system sensitive to classical (intensity) noise. For sufficiently large imbalance or in the case of high optical power this noise becomes a dominant contribution. Second, the balancing itself may fluctuate.

Finally, the linear term is the light shot noise. Obviously we want this last term to be greater than the other two over largest possible photon number per pulse. Apart from the full fit we also plot each of the terms separately. This allows us to determine the range of shot-noise-limited operation. We assume the shot noise limit (SNL) is where the linear term crosses the electronic noise level, $8.5 \times 10^{-7} [V^2]$. This occurs at the photon number $3.26 \times 10^6$. Using the measured value of sensitivity, this electronic noise level is equivalent to $1.7 \times 10^3$ photons. If the measurement is to be consistent, this value should correspond to the shot noise. Indeed, $\sqrt{N} = 1.8 \times 10^3$ which, taking into account the uncertainty of the measurement of $S$, indicates a very good agreement.

The above values have been obtained from a 2-sample measurement and they can be improved whenever the separation between pulses permits averaging. Figs. 5.28 and 5.29 show the improvement obtained by averaging over 1 µs and 3 µs. The variance of the electronic noise is reduced to about $2 \times 10^{-7} [V^2]$ and $1 \times 10^{-7} [V^2]$, which correspond to a reduction in SNL to about $8 \times 10^5$ and to $4 \times 10^5$ photons per pulse, respectively. As before using the measured value of the sensitivity, the averaged electronic noise is equivalent to approx. 810 and 580 photons per pulse difference between the two beams. Again, these coincide with $\sqrt{N}$ which equals approx. 890 and 630. To summarize, we have demonstrated that the SNL operation is guaranteed for pulses with photon number down to $3.2 \times 10^6$. Whenever there is enough time to apply averaging, this can be improved. In particular we have shown that averaging over 3 µs reduces the SNL to about $4 \times 10^5$ photon/pulse.

It should be noticed that although these values have been obtained for integration of definite duration, up to 20 µs in the first case and equal to 7.4 µs in the
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Figure 5.28: Total (light + electronic) noise averaged over 1 µs. Lines assignment is as in Fig. 5.28. Only the lowest electronic noise floor (2 mV/div) is retained.

Figure 5.29: Total (light + electronic) noise averaged over 3 µs. Lines assignment is as in Fig. 5.28. Only the lowest electronic noise floor (2 mV/div) is retained.
second, as has been demonstrated the electronic noise grows very slowly with increasing integration time and thus pulses longer than these may be used without increasing the quoted numbers significantly.

Using the quantum efficiency of the Hamamatsu photodiodes S5973, QE = 0.81, 580 photons correspond to 470 electrons, which is more than the 100-200 read from the specifications of the chip A250 (App. B, Fig. 1) but better than the 730 electrons reported by Hansen [93].

It is interesting to see how the output waveform compares to the output of a trans-impedance circuit. During the measurements we used a New Focus 1801 as a reference. This trans-impedance detector has a bandwidth of 125 MHz which implies a sufficiently fast rise time as to monitor the optical pulses we were using. Figure 5.30 shows the response of the balanced detector constructed during this thesis and that of the New Focus 1801 recorded simultaneously. The oscilloscope bandwidth setting was the same in both cases and equal to 600 MHz. The power on each of the detectors was set the same and it can be read from the output of the New Focus detector which was calibrated. Unlike in typical applications, instead of balancing the detector here we send all the light onto only one of the two photodiodes of the balanced detector.

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12 In [99] the authors report to have constructed a photodetector based on the same model of photodiodes (HAMAMATSU S5973) and the same CSA chip (AMPTEK A250, the type of the FET is not specified) as used here and a shaping stage with AMPTEK A275. They claim to have achieved “intrinsic excess noise” of 80 rms electrons. Surprisingly, this is lower than the values given by the A250 data sheet (Fig. 1 of the spec-sheet). They also report on measuring photon shot noise with pulses of $10^6 - 10^7$ photons but do not report measurements using photon numbers less than quoted although based on their extremely low electronic noise it should be possible.
Figure 5.30: Comparison of the response of the balanced detector (a) and the New Focus 1801 (b). In (a) only one of the photodiodes of the balanced detector is illuminated. Black trace in (b) is a numerically computed integral of the measured signal (blue).
ANY experiments that involve atomic spins are intrinsically sensitive to magnetic fields. This does not come as a surprise. In fact, atomic vapor magnetometers belong to the most sensitive devices in the field [100, 101] achieving sub-femtotesla sensitivities [102] reaching or even beating the sensitivity of SQUID\textsuperscript{1} based magnetometers.

As a consequence a good cancellation of any stray magnetic field is required. This cancellation can be either passive or active. Both methods have been used. For example, all the vapor cell experiments by the Copenhagen group [1–3] rely solely on very heavy magnetic screening. While this is effective, it also dramatically reduces optical access. Active compensation techniques have been developed and implemented for instance in the case of some BEC experiments [106].

While an effective cancellation is necessary, having the possibility of adding a controlled external magnetic field makes the experimental setup more versatile enhancing the list of possible experimental scenarios. In particular dynamic control of the atomic state [37] and quantum feedback [10] have been demonstrated in this context.

A standard way of compensating DC stray magnetic fields (of which the Earth magnetic field is usually the most important one) utilizes a DC current driver and a pair of coils in a roughly Helmholtz configuration along each of the three axes. The nulling procedure relies on the observation of the shift in the position of the cloud trapped in the MOT as the field gradient and light detuning are varied. This technique was implemented in the setup at an early stage [15, 16]

\textsuperscript{1}Superconducting Quantum Interference Device, [103–105]
and it was sufficient at that time. In the context of current experiment, we are interested in protecting the ground state spins from precessing while held in the dipole trap for a possibly long time. Precession is caused not only by DC but also AC fields which cannot be nulled passively. We therefore set out to construct an active control system.

In the first section of this chapter a simple experimental method used to detect AC stray magnetic fields is presented. Later, we introduce a prototype compensation circuit and demonstrate nulling of the signal due to atomic motion in one dimension, at the frequency of the mains. We conclude with a short description of the final version of the circuit.

The work presented in this chapter has been conducted in collaboration with Albert Benseny while the electronic design and manufacture of the final version have been conducted by Ricardo Saiz and Francisco Remiro of the ICFO Electronic Workshop. In the course of the measurements we have observed some unrelated albeit interesting dynamics of the atomic cloud in the magneto-optical trap. These observations are presented and briefly discussed in the last section of the chapter.

## 6.1 Stray magnetic field measurements

The basic idea behind this measurement is that any stray field will affect the position of the zero field of the MOT. Since atoms gather around that point, one can acquire information about the AC stray magnetic fields by observing the position of the atomic cloud. This measurement is not conclusive in the sense that there may be other factors making the atoms oscillate around their equilibrium position. However, there exists an important exception where we can with certainty conclude that the oscillations come from a stray magnetic field. This is the case of oscillations at the mains frequency, 50 Hz.

It is natural to expect that most of the stray magnetic fields will be produced by devices supplied from the mains. Such devices may produce magnetic fields at frequencies being harmonics of the mains frequency, however, one expects that the 50 Hz component will dominate. It is of fundamental importance to note that this component will be always phase-locked to the mains. That is, we have a
good reason to expect that as long as the positions in space of all the potential sources of stray field at 50 Hz remain fixed, an active nulling by phase locking to the mains should be effective. Such an active compensation is demonstrated in the next section.

In order to observe the position of the atomic cloud we have slightly modified the imaging system by shifting the fluorescence collecting photodiode off axis, Fig. 6.1. The offset has to be adjusted so that maximum sensitivity with respect to the AC signals on the photodiode is obtained. The power spectrum of the photodiode signal is then observed on a digital oscilloscope. The top trace in Fig. 6.2 shows this signal. Although not the strongest, the 50 Hz component is clearly resolved. The middle trace shows a corresponding spectrum when the prototype Voltage Controlled Current Source (VCCS) has been used to suppress the oscillations at the mains frequency. The amplitude of the nulling magnetic field was found to be 4 mG. The circuit is described in the next section. We discuss the remaining components of the spectrum at the end of this chapter.

6.2 Nulling the 50 Hz

The nulling observed in the middle trace of Fig. 6.2 has been achieved with a VCCS circuit that is shown in Fig. 6.3. The circuit produces a current proportional to the control voltage $I_{OUT} = V_{IN}/R$. The control voltage has been derived from a low-voltage transformer that provides an automatic phase lock

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2It is a modified version of a design by Mazi Hosseini of Siemens Milltronics Process Instruments (http://electronicdesign.com/Articles/Index.cfm?AD=1&AD=1&ArticleID=9018). The original design supplied current of single polarity only.
Figure 6.2: The power spectrum of the photodiode signal in Fig. 6.1. Top 50 Hz not nulled, middle 50 Hz nulled, bottom reference (no atoms). The vertical scale is the same in all three traces.
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to the mains. A phase-shifting module circuit has been used between the transformer and the VCCS. The output current of the VCCS drove a pair of compensation coils.

![Diagram of the prototype of the Voltage Controlled Current Source.](image)

**Figure 6.3:** The prototype of the Voltage Controlled Current Source.

Once the phase and the amplitude of the control voltage have been adjusted so that the 50 Hz component disappeared, the nulling remained unaltered after switching the supply off and back on. This confirmed that the observed 50 Hz peak was indeed synchronized with the mains. However, the fact that this peak disappeared from the spectrum does not mean that the stray field has been compensated perfectly in this case. The pair of compensation coils used in this example only produced a field along one of the two directions perpendicular to the optical axis in Fig. 6.1 and as such could not compensate stray fields in the other perpendicular direction (vertical in this case). The reason why the compensation appeared so good lies in the fact that the setup in Fig. 6.1 does not distinguish between these two directions, hence it is possible to compensate the photodiode signal due to the motion in one direction by inducing motion in the other. Effectively, the fluorescence light will then be focused in a point that traces an ellipse around the photodiode at the frequency of the mains. If the eccentricity of the ellipse is set so as to take into account the strengths of the stray fields along the two directions, the photodiode signal will be zero. This coupling between the two directions could be removed by using a quadrant photodiode which we did not have at our disposal. We would like to emphasize that this does not invalidate the overall conclusion that the prototype circuit does what it has been designed
to do.

Apart from the peak at 50 Hz, many more features are present in the spectra Fig. 6.1. In particular, there is a pronounced peak at around 40 Hz and there are visible many harmonics of 50 Hz. The former will be discussed in the next section, here we focus on the harmonics.

First, it has to be noticed that most of the signals at frequencies being multiples of 50 Hz are absent in the reference trace recorded without atoms. Second, it is unlikely that stray magnetic fields at harmonic frequencies are stronger that that at the fundamental. Thus a naive conclusion one could draw would be that atomic motion is a nonlinear function of the stray field. This argument can be discarded by comparing the amplitude of the field necessary to compensate the 50 Hz (∼1 mG) with the applied quadrupole field (∼10 G). Yet another possibility would be that the atomic motion is more sensitive to high-frequency fields. However, there is no apparent reason for that to be the case.

The most likely cause of the harmonics and their high amplitudes lies in the very method of measurement. It is easy to see that the signal of the missaligned photodiode (Fig. 6.1) will be a nonlinear function of the position of the source of light, the atoms, and as such it will contain components at harmonic frequencies. While it is true that these harmonic signals should disappear when the atomic motion at the fundamental frequency 50 Hz is removed, one has to keep in mind that the detection setup is not able to differentiate between motions in perpendicular directions. In case of an elliptical orbit described earlier in this section, the peak at 50 Hz will disappear, but the harmonics may remain. Again it should be stressed that despite these imperfections, the implemented method is sufficient for tests of our field control unit. In a case when a more thorough analysis is necessary a quadrant photodiode could be used. Ideally, simultaneous detection in two perpendicular directions would facilitate direct observation of the atomic motion in all three dimensions.

Encouraged by the promising performance of our prototype we decided to build a full three-axis control system that apart from allowing for the compensation of stray fields would also provide a way to apply an arbitrary magnetic field with a reasonably large magnitude. The block diagram and the circuit schematic are included in App. C. The device consists of three independent units each driving a separate pair of coils. Every unit has a low frequency input and another, high
speed input.

The low frequency input has been designed for nulling the 50 Hz component and for that reason it includes a low voltage transformer as a signal source phase-locked to the mains. However, it also can be driven from an external signal source. The phase can be adjusted to an arbitrary value between 0 and $2\pi$ whereas the amplitude range is from 0 to 5 mA which corresponds to 25 mG. This is about 6 times more than the current used in the last section where the nulling was demonstrated leaving a large margin for the future.

The high-speed input is DC-coupled and allows to produce an arbitrary waveform magnetic field. The amplitude range of the current is from -200 mA to 200 mA. With our 1 mH coils this provides a magnetic field of up to 1 G with a measured rise-time below 40 $\mu$s, limited by the output voltage.

### 6.3 MOT dynamics

In this section we briefly report some observations of self-induced MOT dynamics and argue that they are responsible for the large peak at 40 Hz in Fig. 6.1. This peak is of different nature than the others. It is definitely caused by atoms and it is not related to mains frequency, 50 Hz. Also, it is not just an electromagnetic interference like the feature at around 230 Hz.

Dynamic instabilities in magneto-optical traps are commonly observed. In our MOT they usually produce signals so large that spectra like those in Fig. 6.1 are completely obscured. In fact to observe the spectra of Fig. 6.1 the cooling light had to be detuned by -2.5 $\Gamma$. On the other hand MOT dynamics can produce fascinating, complex signals that deserve a brief discussion here.

Figure 6.4 shows a record of the fluorescence signal during the loading of the MOT. During the first few seconds the signal just grows steadily, however, later oscillations settle in. The power spectra of these oscillations at various time intervals are shown in Fig. 6.5.

Several observations can be made. First, there exists one strong discrete compo-

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3The field/current conversion factor is not exactly the same for all three coil pairs because their geometries are not the same. The pair used in the last section has conversion factor of 5 mG/mA and an inductance of about 1 mH.
Chapter 6: Magnetic field control

Figure 6.4: Fluorescence signal of the MOT loading.

In the spectrum a component whose position fluctuates between 150 and 200 Hz. Second, the entire spectrum is confined to two regions, one centered at the aforementioned peak and the other around 300 Hz. Third, the spectrum dynamically changes but always in such a way that the second observation remains true. It should be pointed out that none of the specific frequency components present in Fig. 6.5 corresponds to the single-atom oscillation frequency in the potential of the magneto-optical trap. In fact, substituting our experimental parameters in Eq. 3.1.5 yields \( \omega_0 \approx 1 \text{ kHz} \).

Paradoxically, we do not observe any discrete spectral features below 100 Hz. Thus it could appear, that the 40 Hz peak in Fig. 6.1 is not related to the signals discussed here. However, spectra in Fig. 6.1 have been recorded with a far-detuned MOT, whereas those in Fig. 6.5 correspond to a MOT in its normal operation. It is an everyday experience that the character of the MOT fluctuations is very sensitive to detuning, typically both frequency and amplitude decrease with increased (negative) detuning. We believe that this is the reason why the peak in Fig. 6.1 appears at a lower frequency.

The complex collective behavior of the atomic ensemble in the magneto-optical trap leading to ring-like and similar spatial distributions of atoms has been ob-
**Figure 6.5:** Power spectra of the fluorescence record in Fig. 6.4 at various representative time intervals, 1–2 s, 5–6 s, 6–7 s, 7.5–8.5 s, 9–10 s and 17–18 s, respectively.
observed early [107] [108] and attributed to long-range repulsive interactions that result from multiple photon scattering (radiation trapping) in a dense cloud [109] or to the radiation pressure due to missaligned beams [110]. More recent studies using a quadrant-photodiode [111] have shown that in a trap with retroreflected beams the so called shadow effect can result in slow, stochastic or fast, deterministic dynamics leading to spectra resembling those in Fig. 6.5. The shadow effect refers to the atomic ensemble blocking a portion of the three primary beams which implies an intensity imbalance between the primary beams and the retroreflected ones. This also provides a feedback mechanism. It has been pointed out that fluctuations are produced even with perfectly aligned beams. Finally, Labeyrie and co-workers [112] have shown that collective instabilities due to multiple photon scattering in a large 6-beam MOT display features typical of supercritical Hopf bifurcation and compared the underlying dynamics to those occurring in stars where the competition between the gravity and the radiation pressure also lead to instabilities.
This chapter is to some extent independent. It presents absorption spectra of atoms held in a dipole trap. Because the probe used in this case is resonant, the measurements are easier than those reported in Chap. 8 that employ a detuned probe. In some sense these measurements have served as a test of our ability to directly observe the atom-probe interaction in our dipole trap. In a broader context, the absorption spectra of dipole-trapped cold atoms are interesting in their own right. For example, it is interesting to compare these traces with typical saturation-absorption signals. The destructive effect of a resonant probe has also been observed.

Unlike in Ch. 8, here we are only interested in moving the atomic populations between the ground hyperfine levels, $F = 1$ and $F = 2$. This is accomplished by using only the cooling and the repumping light of the MOT. Given a long and thin atomic sample stored in our dipole trap, the main problem for spectroscopic measurements is that the optical depth, although high on the axis of the sample, drops rapidly as the distance from the axis increases. In this situation one would make best use of the available optical depth by using a collimated beam of a size much smaller than the size if the sample propagating along the sample axis. However, this solution is not very practical in the current setup. Instead, the precision probe whose waist is matched to that of the sample is used. Obviously, this provides more signal than a small collimated beam but there is a price to pay. The portion of the beam that has passed relatively far from the axis and therefore has only weakly interacted with the atoms does not carry much signal yet it contributes noise. Thus the relative amount of light absorbed by the atoms is small and its detection in the presence of noise is a challenge. It is an additional
complication that the dipole trap is shallow and thus the probe intensity must be kept low reducing the available signal.

![Figure 7.1: Setup used for spectroscopy measurements of a MOT and a dipole trap. The round shape depicts atoms in the MOT, the oval shape depicts atoms in the dipole trap.](image)

### 7.1 Photodetector circuit

The two types of noise involved here are the classical laser intensity fluctuations and the light shot noise. In order to eliminate the first one we have used a balanced configuration with a reference beam bypassing the sample. The probe and the reference intensities are then compared on a photodetector that has been built in house purposely for this application. It employs a simple resistive load with a low-noise voltage follower. The typical drawback of this kind of design is its low bandwidth. In this application it is actually not a drawback, on the contrary, it is desired to cut the bandwidth precisely so that the spectroscopic signal is retained while the obscuring noise is removed. The necessary bandwidth is given by the scan frequency and the circuit is adjusted by varying the load resistor. Load resistance of up to 40 MΩ has been used producing good results with a moderately fast scan.

### 7.2 MOT spectroscopy

As a preliminary step we have recorded spectroscopy traces from an operating magneto-optical trap. Since the MOT is much larger and contains many more atoms this experiment is a good test bed for later applications to dipole trapped atoms. Figure 7.3 shows simultaneously recorded absorption trace of the MOT.
and a saturated absorption trace from a vapor cell. Only the $F = 2 \rightarrow F'$ man-

ifold has been observed because the presence of the MOT repumper effectively empties the $F = 1$ ground level. The signal is a single scan record and it clearly resolves the three real transitions.

## 7.3 Dipole trap spectroscopy

Because of the significantly smaller number of atoms and also the small size of the sample, obtaining high signal/noise absorption spectroscopy signals is by far
more difficult than in the case of the MOT. In Fig. 7.4 a single scan trace of dipole

![Simultaneously recorded transmission spectra of the F = 2 manifold of dipole trapped atoms (upper blue) and a vapor cell (lower green).](image)

Figure 7.4: Simultaneously recorded transmission spectra of the $F = 2$ manifold of dipole trapped atoms (upper blue) and a vapor cell (lower green).

trapped atoms is displayed. Atoms are forced into $F = 2$ by deliberately switching the MOT repumper off only after the cooling beams have been switched off. Note that this time only the closed transition $F = 2 \rightarrow F' = 3$ is clearly resolved, the other two $F = 2 \rightarrow F' = 1$ and $F = 2 \rightarrow F' = 2$ being undetectable. The reason for this is that unlike in the MOT the pumping effect of the probe cannot be ignored. In the MOT the atoms that spontaneously decay to $F = 1$ are brought back to $F = 2$ by the repumping light. Here, in the absence of any repumping mechanisms, they are lost. This gradual optical pumping into $F = 1$ caused by the probe severely reduces the available signal and as a consequence only the closed transition, $F = 2 \rightarrow F' = 3$, can be detected. An interesting feature can be observed when the dipole trapped atoms transmission trace is inverted and then shifted and scaled (both in the vertical direction) as in Fig. 7.5. A shift of the resonance peak is visible. This is a blue shift as indeed expected from a red detuned dipole trap. The maximum value of the shift is given by the algebraic difference\(^1\) of the shift of the excited state and that of the ground state and is about 15 MHz (Eq. 3.4.8 and [79]). However, such a large shift is only experienced by the atoms occupying the very bottom of the trap. All others experience

\(^1\)Note that the signs of the two shifts are opposite.
a smaller shift. As a consequence, the measured value will always be smaller than the quoted number and, in addition, the linewidth will be broadened. The value of the light shift extracted from Fig. 7.5 is $4.6 \pm 3.5$ MHz, which lies within the expected range. The measured FWHM linewidth is $9.1 \pm 0.6$ MHz. Since the Doppler shift at temperature around 50 µK is below 0.2 MHz we explain the observed broadening as a combined effect of the aforementioned mechanism, the power broadening mechanism and the technical noise of the measurement.

Observing spectroscopy signals from dipole trapped atoms prepared in the $F = 1$ has been even more difficult. The trace in 7.6 has been obtained with just about 90 nW probe. This time the repumper is switched off first and the cooling light eventually pumps the atoms into $F = 1$. Again, the reason why the weak transition $F = 1 \rightarrow F' = 0$ appears strongest is that it is the only closed transition which does not cause an optical pumping into the $F = 2$.

We have also tried combining multiple pumping schemes on the same sample. Figure 7.7 shows a spectroscopy trace with the probe tuned to $F = 2 \rightarrow F'$. The atoms are first pumped into $F = 1$ with the cooling laser. The spectroscopy
Figure 7.6: Simultaneously recorded transmission spectra of the $F = 1$ manifold of dipole trapped atoms (upper blue) and a vapor cell (lower green). The arrows mark the transitions: from left to right $F = 1 \to F' = 0$, $F' = 1$, $F' = 2$. 

\[ \times 10^{-3} \]
Figure 7.7: $F = 2$ manifold transmission spectrum of dipole trapped atoms. The spectrum on the left is for atoms pumped into $F = 1$ and the one on the right has been measured for the same sample after pumping back into $F = 2$ (see text).
signal is then very weak. We next use the repumper light to bring the atoms back to $F = 2$. This time the large amplitude of the spectroscopy signal is restored.
Chapter 8

Polarization rotation measurements

In sections 2.4.6 and 2.4.7 of chapter 2 it was explained how a measurement of macroscopic polarization rotation caused by an imbalance between the Zeeman state populations can be used to determine the effective coupling constant, $\kappa_{\text{eff}}^2$. Measurements of this type are reported in the present chapter.

For small rotations, the rotation angle of the polarization plane is given by (Eq. 5.1.6)

\[ \theta_E = \frac{N_{\text{ph},3\pi/4} - N_{\text{ph},\pi/4}}{2N_{\text{ph}}} \]  \hspace{1cm} (8.0.1)

where the numerator stands for the photon number difference as measured in the basis tilted by 45° with respect to the initial polarization. $N_{\text{ph}}$ is the total number of photons in the pulse.

An example of the detector output voltage in a typical polarization rotation measurement is shown in Fig. 8.1(a). Here the optical signal is a train of pulses, each 4 µs long with a period of 26 µs. As explained in Sec. 5.4.2, Ch. 5, measurements of unipolar pulse sequences result in a voltage pile-up at the output of the detector which in turn causes the relatively slow, $\tau = 300$ µs (Sec. 5.4.4), to contribute a non-negligible drop of the signal during the pulse itself.

Apart from the exponential, $\tau = 300$ µs, discharge visible between the pulses, another effect can be observed in Fig. 8.1(a). After some initial growth the signal reaches a maximum and subsequently the overall tendency reverses; we observe a decrease. This decrease is not related to the discharge of the capacitor of the charge sensitive amplifier. It is caused by the input, $\tau = 1$ ms high-pass filter (Sec. 5.4.4).
In order to distill the real values of the voltage steps and hence the polarization rotation angle from the measured signals the above effects have to be accounted for. This is implemented in the algorithm described in the following section.

### 8.1 Data analysis

The trace in Fig. 8.1(b) is a magnified portion of the one in 8.1(a). For clarity no line has been plotted between the data points.

The first step of the data analysis consists in identifying the samples defining the beginning and the end of each pulse. These are marked by black circles and black stars, respectively. They occur at \( t = t_1^i \) and \( t = t_2^i \) and the measured values are denoted by \( V_1^i \) and \( V_2^i \), where \( i \) is the running index of the pulse within the train.

Next step is to delimit the portion of the signal lying between consecutive pulses. The relevant samples are denoted by a '×' and a '+' marks. They happen at \( t = t_3^i \) and \( t = t_4^i \) and the measured values are denoted by \( V_3^i \) and \( V_4^i \). In this case it is not important to mark the intervals precisely. The essential point is that the '×'- and the '+'-marked samples are taken between the actual pulses and not during a pulse.

The basic idea now is to approximate the portions of the signal between pulses by straight lines. This is an excellent approximation as long as the period of the pulse train is much shorter than the shortest of the intrinsic time constants of the detector, i.e. 300\( \mu \)s. By fitting a line

\[
V = a^i \times t + b^i
\]  

(8.1.1)

to the data set between \( t = t_3^i \) and \( t = t_4^i \), we find the coefficients \( a^i \) and \( b^i \) for every inter-pulse interval. Using these coefficients the values of the linear fits at \( t_1^i \) to \( t_4^i \) are computed, \( U_1^i = a^i \times t_1^i + b^i \) and so on. Instead of taking the difference \( V_2^i - V_1^i \), the actual values corresponding to the integrals over each optical pulse are computed from the following formula

\[
\Delta U^i = U_2^i - U_1^i - \frac{a^{i-1} + a^i}{2} \times (t_2^i - t_1^i).
\]  

(8.1.2)

The first part of 8.1.2, \( U^i = U_2^i - U_1^i \), replaces the direct difference, \( V_2^i - V_1^i \), giving a better estimate of the voltage jump due to the optical pulse. The remainder
Figure 8.1: A typical polarization rotation signal when the probe consists of a series of pulses. (a) detector output voltage, (b) magnified data from (a) showing the samples; the ones used of special importance in the data analysis are marked (see text), (c) pulse integrals computed according to the algorithm given in the text.
represents a correction that allows to cancel the effects of electronic decays in the system, \( \frac{a^\dagger_{-1} + a^\dagger_1}{2} \) being the average of the slopes before and after the pulse.

Following this algorithm the data in Fig. 8.1(b) can be represented by a bar plot, as in Fig. 8.1(c). The height of the bars is given by Eq. 8.1.2 while their width has been set to be equal to the pulse duration to ensure an intuitive visual effect. The bar plots in this and the next chapters have been obtained in this way. Additionally, the quantities \( \Delta U_i \) plotted in Fig. 8.1(c) directly in Volts, in subsequent plots have been expressed in photon number using the detector gain factor, 0.55 \( \mu V/\text{photon} \) (Sec. 5.4.5). Knowing the total number of photons per pulse, the polarization rotation angle can then be calculated from Eq. 8.0.1.

8.2 Population imbalance

In this section the results of polarization rotation measurements using the technique described in Sec. 2.4.6 (Ch. 2) are presented. The atoms are optically pumped into \( |F = 1, m = -1 \rangle \) or \( |F = 1, m = 1 \rangle \) with a single \( \sigma^- \) and \( \sigma^+ \) polarized pulse respectively, tuned to \( |F = 1 \rangle \rightarrow |F' = 1 \rangle \) as described in Sec. 4.1.1 of Ch. 4.

For measurements reported in this section, the duration of the optical pumping pulse has been set to 2 \( \mu s \) and the power is adjusted such that the intensity on the optic axis is roughly equal to the saturation intensity. The pulse duration had been optimized so that the rotation signal was maximum. The existence of this maximum may be related to an imperfect polarization state of the pumping light. As a consequence, the \( |F = 1, m = -1 \rangle \) and \( |F = 1, m = 1 \rangle \) are not exact dark states for the pumping light. Because of this there is always a chance that an atom once pumped into the desired state gets again excited and decays into one of the \( |F = 2 \rangle \) states. As no repumping light that would bring those atoms back into \( |F = 1 \rangle \) has been implemented, they are lost for the process.

As mentioned in Sec. 4.1.1 of Ch. 4, setting the polarization of the pumping light is complicated by the presence of the dichroic cube, DC1 in Fig. 4.1 which introduces a different phase shift for p- and s-polarized components thus scrambling the polarization of the reflected beam. In order to tackle the problem, the polarization state in front of the collimating lens L4 (Fig. 4.1) is analyzed with
a half-wave plate and a polarizing beam splitter. Since the effect of a half-wave plate on a perfectly circularly polarized light is just to switch the helicity, placing a PBS after the wave plate should give a balanced beam pair irrespective of the wave plate angle. Any variation in the splitting ratio as the wave plate is rotated indicates a deviation from circular polarization. Using this method the angles of the waveplates HW3 and QW2 (Fig. 4.1) that produce best $\sigma^{-}$ polarization have been found to be $53^\circ$ and $168^\circ$, respectively. For $\sigma^{+}$ the corresponding angles are $131^\circ$ and $180^\circ$. The purity of the circular polarization expressed in terms of the Stokes parameters has been measured to be $S_z/S_0 = 0.87$ and $S_z/S_0 = 0.86$.

Comparing the first of the two angle pairs found above with the one maximizing the degree of $\sigma^{-}$ polarization in front of the dichroic cube DC1, $50^\circ$ and $176^\circ$ for HW3 and QW2 respectively, it may seem a minor correction. However, for the latter pair the fraction $S_z/S_0$ as measured after DC1 is only 0.30.

The $\sigma^{-}$ and $\sigma^{+}$ polarization states have been identified by replacing the analyzing half-wave plate with a quarter-wave plate. A quarter-wave plate transforms a circular polarization into linear polarization at $45^\circ$ or $135^\circ$ to the fast axis of the wave-plate depending on the helicity.

Figure 8.2 shows bar plots obtained from the measured signal as explained in the preceding section. For these measurements the number of atoms was about $N_{at} = 1 \times 10^5$ and a 1 G, guiding magnetic field was applied along the the $z$-axis to prevent atoms from precessing around arbitrary directions due to stray fields. The probe is a train consisting of 32 pulses each 4 $\mu$s long with a 26 $\mu$s period. The probe power is 4 $\mu$W and its detuning from $|F = 1 \rangle \rightarrow |F' = 0 \rangle$ transition, $\Delta = -150$ MHz. The linearity of the probe polarization is verified in a similar fashion as that of the pump, a half-wave plate and a PBS are placed in front of L4. A degree of linear polarization given by $S_x/S_0 = 0.994$ has then been obtained by adjusting the HW2 and QW1 by small angles.

The measured signal has not been used directly, rather an average over 10 experimental realizations has been computed. Directly after this measurement a reference trace without atoms has been recorded to cancel any residual imbalance on the detector. This reference trace is subtracted from the actual rotation mea-

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1The reference from which the angles are measured is determined by convenience and the absolute values of the angles do not have any meaning. The angle pair $50^\circ$ and $176^\circ$ corresponds to a circular polarization after QW2 and in front of DC1.
Figure 8.2: Polarization rotation measurements. Atoms have been pumped into $|F = 1, m = -1\rangle$ (a), and $|F = 1, m = +1\rangle$ (b). The horizontal photon shot noise lines have been scaled by a factor $\sqrt{10}$ to account for the averaging.
CHAPTER 8: POLARIZATION ROTATION MEASUREMENTS

measurement and in Fig. 8.2 appears the difference. In addition to the experimental data the photon shot noise level has been plotted for comparison. The little, random bars appearing before the actual pulse train have been plotted intentionally. They are the outcome of applying the data analysis procedure described previously when no light is present and correspond to the electronic noise of the detection system.

The plots in Fig. 8.3 show results obtained with the probe train extended to 256 pulses. Other parameters are unchanged. For comparison, in Fig. 8.4, the detector signal\(^2\) measured with a probe train consisting of 32 pulses, each lasting 4 \(\mu\)s, and a 10 ms period, has been plotted. There is a clear difference between these two measurements. In the first case the signal disappears entirely after a couple of milliseconds whereas in Fig. 8.4 it persists for more a 100 ms. This is a clear evidence of the destructive effect of the probe. Moreover, it indicates that the prepared quantum states have a long decoherence time, comparable to that of the trap itself.

Using Eq. 8.0.1 with the probe pulse power and duration as quoted above, the polarization rotation experienced by the initial pulses of the trains shown in Fig. 8.2 is roughly 0.2 mrad. Equation 2.4.25 then yields \(\kappa^2_{\text{eff}} = (\theta_E)^2 4N_{\text{ph}} / N_{\text{at}} \approx 10^{-4}\)

By considering Eq. 2.4.28 of Ch. 2 we can conclude that the observed value of the effective coupling constant is not sufficient to expect a significant reduction in the variance of the pseudo-spin component, \(\hat{J}_z\) and consequently it is not likely to provide measurable spin squeezing. In fact, initial trials have not provided evidence for reduction of atomic noise. In the next section it is shown that there are reasons to suspect that the low measured value of \(\kappa^2_{\text{eff}}\) is related to inefficient state preparation and that by making the optical pumping more efficient the coupling strength can be significantly increased.

We conclude this section with the observation that although the measured coupling is not satisfactory at the moment, it allows for measurements of polarization rotation with a sensitivity corresponding to \(10^4\) atoms in a single shot measurement. Averaging over multiple traces can be used in order to bring the sensitivity to an, in principle, arbitrary value (Fig. 8.2).

\(^2\)Here the output voltage has been plotted directly. For such long pulse periods a good visual effect is obtained in this way and if exact values are not important the data analysis algorithm can be abandoned at this time.
Figure 8.3: Polarization rotation measurements with a long probe pulse train. 
(a) $|F = 1, m = -1\rangle$, (b) $|F = 1, m = +1\rangle$. 
Figure 8.4: Polarization rotation measurements with a long probe pulse period. (a) $|F = 1, m = -1\rangle$, (b) $|F = 1, m = +1\rangle$. 
8.3 Axial magnetic field

This section presents the results of measurements of the polarization rotation induced by an applied magnetic field according to the description given in Sec. 2.4.7 of Ch. 2. As explained therein, this technique provides an upper bound for the coupling constant, $\kappa_{\text{eff}}^2$, that can be used to verify the efficiency of the state preparation.

Figure 8.5 shows polarization rotation signals obtained with a sample of about $2 \times 10^5$ atoms and a probe consisting of $1.2 \times 10^8$ photons. The detuning has been set to $\Delta = -2\pi \times 350$ MHz. In this case atoms are not optically pumped and we assume a roughly uniform distribution among the three Zeeman states resulting in $N_+ \approx N_- \approx 1/3 \times 2 \times 10^5 \approx 7 \times 10^4$. The three traces in Fig. 8.5 correspond to rotation observed as the magnetic field is varied in steps of 0.5 G. The nonvanishing rotation signal when the applied field is zero is attributed to a non-zero stray field.

Substituting the parameters above and the measured rotation angles to Eq. 2.4.33 (see also Eq. 8.0.1) yields $\kappa_{\text{eff}}^2 \approx 1$. Despite the larger detuning (-350 MHz versus 150 MHz) this is by far greater than the value estimated in the previous section. This large difference indicates that in the measurements reported in Sec. 8.2 the number of atoms effectively interacting with the probe was in fact significantly smaller than anticipated. This is attributed to inefficient state preparation which takes atoms into $|F = 2\rangle$ where they become invisible to the probe.

The much greater value of $\kappa_{\text{eff}}^2$ obtained in this section strongly suggests that improvement in the observed atom-light coupling is possible and that in order to achieve it the first step should consist in increasing the efficiency of the optical pumping. In particular, implementing an optical repumping laser that will prevent loss of atoms into $|F = 2\rangle$. The results of this section also confirm the usefulness of the presented method. Being independent of the population distribution among the Zeeman states, it allows for a verification of the efficiency of the optical pumping.
Figure 8.5: Polarization rotation observed as the applied magnetic field is varied, (a) $B_z = -1$ G, (b) $B_z = -0.5$ G, (c) $B_z = 0$ G.
Chapter 9

Conclusion

The main theme of this theses has been the construction and characterization of a system to study light-atom interaction for future implementations in continuous variable quantum information protocols. For this application an existing MOT setup has been extended to include a far off-resonant dipole trap. The principal parameters of the trap have been calculated and measured. Remarkably, a lifetime exceeding $\sim 30$ s has been measured. A typical number of trapped atoms has been $\sim 10^5$. Simulations based on a simple model have indicated that by loading colder atoms it should be possible to further improve this figure.

One of the principal tasks of this project was a construction of a balanced shot-noise limited photodetection system. To that end a new approach has been adopted. In contrast to some previous implementations, the design exploits only a charge-sensitive preamplifier stage without a pulse-shaping section that has been replaced with a simple numerical algorithm implemented in the process of data analysis. This removes some of the inconveniences of using a particle-detector type devices in optical domain with $\mu$s-pulses and allows for a greater flexibility with respect to the optical signal. As a bottom line, shot-noise limited operation has been demonstrated that spans 3 orders of magnitude and down to $\sim 10^5$ photons per pulse.

The capability of the setup to prepare desired initial states of the atoms and to control the interaction of the probe and the trapped atomic ensemble, has been verified in three experiments. As an initial test, the population of the ground hyperfine levels has been manipulated and the outcome of this procedure has been recorded in a dedicated absorption spectroscopy setup.
The other two experiments have been performed with an off-resonant probe. The goal of these efforts was to assess the experimentally available strength of the probe-ensemble interaction characterized by the parameter, $\kappa^2$. In the first attempt, polarization rotation due to population imbalance has been observed and measured. These measurements showed that the amount of coupling currently available in the setup is insufficient to implement quantum information tasks. The most likely cause of this was poor efficiency of the state preparation.

The aim of the third experiment has been to verify the above conjecture. In this case external magnetic field has been applied and the resulting polarization rotation has been measured. This effect is independent of the population imbalance, thus by comparison with the results of the previous method, it has been possible to distinguish between inefficient state preparation and poor geometrical matching of the probe and the ensemble. Indeed, the measurements indicate that $\kappa \sim 1$ is a realistic number in the current setup, thus confirming the earlier guess. This strongly suggests that the current method of optical pumping requires further work. In particular, it is expected that the state preparation procedure would greatly benefit from implementing a “repumping” field that minimizes loss of atoms to $F = 2$.

Another observation that brings optimism is the long lifetime of the atomic polarization. It turned out that the limitations are only the lifetime of the trap and the destructive effect of the probe.

As a concluding remark I would like to express my belief that implementing the suggested improvements, the setup should prove effective in demonstrating the effects of light-atom interaction at the quantum level. Implementing quantum-information tasks, like quantum memory will then be feasible. Production of spin-squeezed states will be an important milestone towards this goal.
APPENDIX A

Irreducible representation of the interaction Hamiltonian

This presentation follows closely that of Dr. de Echaniz [113] which in turn is based on a derivation by Geremia et al. [35] that appears also in [88].

Hamiltonian

Consider the standard dipole Hamiltonian describing the interaction of a single atom and a single mode of light

\[ \hat{H}_I = -\hat{d} \cdot \hat{E}. \]  

(A.0.1)

The dipole moment operator \( \hat{d} \) can be decomposed into its lowering and raising components

\[ \hat{d}(-) = \sum_{f,m,f',m'} |f,m\rangle \langle f,m| \hat{d} |f',m'\rangle \langle f',m'|, \]

\[ \hat{d}(+) = \sum_{f,m,f',m'} |f',m'\rangle \langle f',m'| \hat{d} |f,m\rangle \langle f,m|. \]  

(A.0.2)

A shown in Sec. 2.2, the electric field operator \( \hat{E} \) can be cast in a similar expression

\[ \hat{E} = \hat{E}^{(+)} + \hat{E}^{(-)} \]  

(A.0.3)

where \( \hat{E}^{(+)} \) and \( \hat{E}^{(-)} \) are the positive and the negative frequency parts

\[ \hat{E}^{(+)} = \sqrt{\hbar g} \hat{a} \hat{e}, \]

\[ \hat{E}^{(-)} = (\hat{E}^{(+)})^\dagger = \sqrt{\hbar g} \hat{a}^\dagger \hat{e}^*. \]  

(A.0.4)
APPENDIX A: IRREDUCIBLE REPRESENTATION OF THE INTERACTION

HAMILTONIAN

In A.0.4 we have used the circular polarization basis defined in Eq. 2.2.6.

The rotating wave approximation and perturbation expansion techniques can be used to write Eq. A.0.1 in the following form [27, 114, 115]

\[ \hat{H}_I = - \sum_{f'} \hat{E}^{(-)} \cdot \hat{\alpha}_{f,f'} \cdot \hat{E}^{(+)}, \] (A.0.5)

with the polarizability tensor \( \hat{\alpha}_{f,f'} \)

\[ \hat{\alpha}_{f,f'} = - \frac{\hat{d}^{(-)} \hat{d}^{(+)}}{\hbar} \frac{\Delta_{f,f'}}{\Delta^2_{f,f'} + \Gamma_f^2 / 4} e^{-q' q} \] (A.0.6)

where \( \Delta_{f,f'} \) is the detuning of the probe from the transition \( |f, m\rangle \rightarrow |f', m'\rangle \) and \( \Gamma_f \) is the decay rate of the state \( |j', f', m'\rangle \).

**Dipole moment**

The dipole moment matrix element can be factorized into the "geometrical" coefficient and the ("physical") reduced matrix element with the help of the Wigner-Eckart theorem [18]

\[ \langle f', m'|\hat{d}_q|f, m \rangle = (-1)^{f'-m'} \left( \begin{array}{cc} f' & 1 \\ -m' & q \end{array} \right) \langle f'\|\hat{d}\||f \rangle. \] (A.0.7)

where \( \left( \begin{array}{cc} f' & 1 \\ -m' & q \end{array} \right) \) is the Wigner 3-j symbol. The reduced matrix element \( \langle f'\|\hat{d}\||f \rangle \) can then be simplified further

\[ \langle f'\|\hat{d}\||f \rangle = (-1)^{j'+i+f+1} \sqrt{(2f+1)(2f'+1)} \left\{ \begin{array}{c} j' & f' & i \\ f & j & 1 \end{array} \right\} \langle j'\|\hat{d}\||j \rangle. \] (A.0.8)

where \( \left\{ \begin{array}{c} j' & f' & i \\ f & j & 1 \end{array} \right\} \) is the 6-j symbol and the reduced matrix element \( \langle j'\|\hat{d}\||j \rangle \) is given in terms of experimentally measurable quantities \( \Gamma_f \) and \( \lambda \)

\[ |\langle j'\|\hat{d}\||j \rangle|^2 = (2j' + 1) \frac{3e_0 \hbar \Gamma_j \lambda^3}{8\pi^2}. \] (A.0.9)
Using the above expressions in A.0.6 yields

\[ \&_{f,f'} = \sum_{m,m',m''} \sum_{q,q'} (-1)^{f+j+2i+2f'+2m'-(2f+1)(2f'+1)} \]
\[ \times \{ \begin{array}{ccc} j' & f' & i \\ f' & j' & 1 \end{array} \} \{ \begin{array}{ccc} j & f & i \\ f & j & 1 \end{array} \} |\langle j' \| \hat{a} \| j \rangle|^2 \]
\[ \times \left( \begin{array}{cc} f & 1 \\ -m'' & -q' \\ m' \end{array} \right) \left( \begin{array}{cc} f' & 1 \\ -m' & q \end{array} \right) \times \frac{\Delta_{f,f'}^2}{\Delta_{f,f'}^2 + \Gamma_{f'}^2 / 4} |f, m'\rangle \langle f, m| e_q e_q^* \]

which satisfies the selection rules \( m' = m + q = m'' + q' \). Then using

\[ \{ \begin{array}{ccc} j & f & i \\ f' & j' & 1 \end{array} \} = \{ \begin{array}{ccc} j' & f' & i \\ f & j & 1 \end{array} \} \]

(A.0.11)

and the fact that \( 2f + 2f' \) is an even integer yields

\[ \&_{f,f'} = \sum_{m,m',m''} \sum_{q,q'} (-1)^{f+j+2i-2m'} (2f+1)(2f'+1) \]
\[ \times \{ \begin{array}{ccc} j' & f' & i \\ f & j & 1 \end{array} \}^2 (2j'+1) \alpha_0 \]
\[ \times \left( \begin{array}{cc} f & 1 \\ -m'' & -q' \\ m' \end{array} \right) \left( \begin{array}{cc} f' & 1 \\ -m' & q \end{array} \right) \times \frac{\Delta_{f,f'}^2}{\Delta_{f,f'}^2 + \Gamma_{f'}^2 / 4} |f, m''\rangle \langle f, m| e_q e_q^* \]

where we have defined

\[ \alpha_0 = \frac{|\langle j' \| \hat{a} \| j \rangle|^2}{(2j'+1)} = \frac{3\epsilon_0 \hbar \Gamma_f \lambda^3}{8\pi^2}. \]

(A.13)

Defining

\[ \alpha'_f = \alpha_0 (-1)^{j' + j + 2i + 2j' + 1} \left( \begin{array}{ccc} j' & f' & i \\ f & j & 1 \end{array} \right)^2 \frac{\Delta_{f,f'}}{\Delta_{f,f'}^2 + \Gamma_{f'}^2 / 4} \]

(A.14)

A.0.12 further simplifies to

\[ \&_{f,f'} = \sum_{m,m',m''} \sum_{q,q'} (-1)^{2m'} (2f+1)(2f'+1) \alpha'_f \]
\[ \times \left( \begin{array}{cc} f & 1 \\ -m'' & -q' \end{array} \right) \left( \begin{array}{cc} f' & 1 \\ -m' & q \end{array} \right) |f, m''\rangle \langle f, m| e_q e_q^*. \]

(A.15)
Spherical tensor decomposition

It is possible to create an irreducible spherical tensor from a linear combination of the projectors $|f, m''\rangle \langle f, m|^{116}$

$$\hat{V}_{Q'}^{(K')} = \sum_{m, m''} (-1)^{f-m} \langle f, m''; f, -m| K', Q' \rangle |f, m''\rangle \langle f, m|,$$  \hspace{1cm} (A.0.16)

where $\langle f, m''; f, -m| K', Q' \rangle$ is the Clebsch-Gordan coefficient, $0 \leq K' \leq 2f$, $K' \in \mathbb{N}$ and $-K' \leq Q' \leq K'$. The inverse relation is

$$|f, m''\rangle \langle f, m| = \sum_{K', Q'} (-1)^{f-m} \langle f, m''; f, -m| K', Q' \rangle \hat{V}_{Q'}^{(K')}.$$  \hspace{1cm} (A.0.17)

Tensors A.0.16 can also be expressed in terms of the spherical angular momentum components according to (up to rank-2)

$$\hat{V}_{Q'}^{(0)} = \frac{1}{\sqrt{2f + 1}} \hat{1},$$

$$\hat{V}_{Q'}^{(1)} = \frac{\sqrt{3}}{\sqrt{(2f + 1)(f + 1)}} \hat{f}_{Q'},$$

$$\hat{V}_{Q'}^{(2)} = \frac{N_2}{\sqrt{6}} (3\hat{f}_z^2 - \hat{f}^2),$$

$$\hat{V}^{(2)}_{\pm 1} = N_2 \hat{f}_\pm (\hat{f}_z \pm \hat{1}/2),$$

$$\hat{V}^{(2)}_{\pm 2} = \frac{N_2}{\sqrt{2}} f_\pm,$$  \hspace{1cm} (A.0.18)

where

$$N_2 = \frac{\sqrt{30}}{\sqrt{(2f+3)(2f+1)(2f-1)(f+1)}},$$  \hspace{1cm} (A.0.19)

and $\hat{1}$ stands for the identity operator. The operators $\hat{f}_z$, $\hat{f}^2$ obey the standard angular momentum operators equations 2.1.1; $\hat{f}_\pm$ is defined as in 2.1.6 if we substitute $\hat{f}$ for $\hat{j}$.

Similarly, given two irreducible tensor operators $\hat{U}_{q'}^{(k')}$ and $\hat{W}_q^{(k)}$ we can form another irreducible tensor operator $\hat{T}_Q^{(k)}$ [117]

$$\hat{T}_Q^{(k)} = \sum_{q, q'} \langle k', q'; k, q| K, Q \rangle \hat{U}_{q'}^{(k')} \hat{W}_q^{(k)},$$  \hspace{1cm} (A.0.20)

the inverse given by

$$\hat{U}_{q'}^{(k')} \hat{W}_q^{(k)} = \sum_{k, Q} \langle k', q'; k, q| K, Q \rangle \hat{T}_Q^{(k)}.$$  \hspace{1cm} (A.0.21)
Substituting $\hat{U}_q^{(k')} = \hat{d}_{q'}^{-}$ and $\hat{W}_q^{(k)} = \hat{d}_q^{(+)}$ with $k = k' = 1$ because we are combining two vectors to form a dyad, we obtain

$$\begin{align*}
\hat{T}_Q^{(K)} &= \sum_{q,q'} \langle 1, -q'; 1, q | K, Q \rangle \hat{d}_{-q'}^{-} \hat{d}_q^{(+)} \\
\text{(A.0.22)}
\end{align*}$$

and the corresponding inverse

$$\begin{align*}
\hat{d}_{-q'}^{-} \hat{d}_q^{(+)} &= \sum_{K,Q} \langle 1, -q'; 1, q | K, Q \rangle \hat{T}_Q^{(K)}.
\text{(A.0.23)}
\end{align*}$$

where $0 \leq K \leq 2$ and $-K \leq Q \leq K$.

### Polarizability

We now apply the results of the last section to the polarizability tensor $\mathcal{\alpha}_{f,f'}$. Using A.0.23 in A.0.6 yields

$$\begin{align*}
\mathcal{\alpha}_{f,f'} &= \sum_{K,Q,q,q'} \langle 1, -q'; 1, q | K, Q \rangle \hat{T}_Q^{(K)} e_{-q}^* e_q^* \\
&= \mathcal{\alpha}^{(0)}_{f,f'} + \mathcal{\alpha}^{(1)}_{f,f'} + \mathcal{\alpha}^{(2)}_{f,f'},
\text{(A.0.24)}
\end{align*}$$

where

$$\begin{align*}
\mathcal{\alpha}^{(K)}_{f,f'} &= \sum_{Q,q,q'} \langle 1, -q'; 1, q | K, Q \rangle \hat{T}_Q^{(K)} e_{-q}^* e_q^*.
\text{(A.0.25)}
\end{align*}$$
This can be expressed more explicitly as

\[
\hat{\alpha}_{f,f'}^{(0)} = -\frac{\hat{T}_0^{(0)}}{\sqrt{3}} (e_0 e_0^* + e_- e_+ + e_+ e_-),
\]

\[
\hat{\alpha}_{f,f'}^{(1)} = \frac{\hat{T}_0^{(1)}}{\sqrt{2}} (e_+ e_+ - e_- e_-)
- \frac{\hat{T}_0^{(1)}}{\sqrt{2}} (e_0 e_0^* + e_0 e_0^*)
+ \frac{\hat{T}_0^{(1)}}{\sqrt{2}} (e_0 e_0^* + e_+ e_+),
\]

\[
\hat{\alpha}_{f,f'}^{(2)} = \frac{\hat{T}_0^{(2)}}{\sqrt{6}} (2e_0 e_0^* - e_- e_- - e_+ e_+)
+ \frac{\hat{T}_0^{(2)}}{\sqrt{2}} (e_0 e_0^* - e_0 e_0^*)
+ \frac{\hat{T}_0^{(2)}}{\sqrt{2}} (e_0 e_0^* - e_+ e_+)
- \hat{T}_0^{(2)} e_- e_+
- \hat{T}_0^{(2)} e_+ e_-.
\]

Using now the definitions of \(\hat{T}_Q^{(K)}\) in Eq. A.0.22 and \(\hat{V}_{Q'}^{(K')}\) in Eq. A.0.17, and following the derivation in Eqs. A.0.10 - A.0.15, we obtain

\[
\hat{T}_Q^{(K)} = \sum_{K',Q'} \sum_{m,m',m''} \sum_{q,q'} (-1)^{f-m-m'-m''} (2f+1)(2f'+1) \alpha_f^{f'}
\times \left( \begin{array}{cc} f & 1 \\ -m'' & -q' \end{array} \right)
\times \left( \begin{array}{cc} f' & 1 \\ -m' & q \end{array} \right)
\times \langle 1, -q'; 1, q | K, Q \rangle \langle f, m''; f, -m | K', Q' \rangle \hat{V}_{Q'}^{(K')}.
\]

We can rewrite this expression in terms of the Wigner 3-j symbols using the identity [117]

\[
\langle f, m; k, q | f', m' \rangle = (-1)^{f-k+m'} \sqrt{2f'!+1} \left( \begin{array}{ccc} f & k & f' \\ m & q & -m' \end{array} \right),
\]

(A.0.28)
to obtain

\[ \hat{T}^{(K)}_{Q} = \sum_{K',Q',m,m',m'',q,q'} (-1)^{f-m-m'-m''+Q+Q'} \times (2f+1)(2f'+1) \sqrt{(2K+1)(2K'+1)} \alpha_f \times (\begin{array}{ccc} f & k & f' \\ m & q & m' \end{array}) (\begin{array}{ccc} k & f & f' \\ m & q & m' \end{array}) (\begin{array}{ccc} f' & 1 & f \\ -m' & q & m \end{array}) (\begin{array}{ccc} f' & 1 & f \\ -m' & q & m \end{array}) (\begin{array}{ccc} 1 & 1 & K \\ -q' & q & -Q \end{array}) (\begin{array}{ccc} f & f & K' \\ m'' & m & -Q' \end{array}) \hat{V}^{(K')}_{Q'} \]  

(A.0.29)

This expression can be simplified by using the following identities [117, 118]

\[ (\begin{array}{ccc} f & k & f' \\ m & q & m' \end{array}) = (\begin{array}{ccc} k & f' & f \\ q & m' & m \end{array}) = (-1)^{f+k+f'} (\begin{array}{ccc} k & f & f' \\ m & q & m' \end{array}) = (-1)^{f+k+f'} (\begin{array}{ccc} f & k & f' \\ -m & -q & -m' \end{array}) \]  

(A.0.30)

and

\[ \sum_{m,m',m'',q,q'} (-1)^{f+f'+f''+k+k'-m-m'-m''-q-q'} \times (\begin{array}{ccc} k & K & k' \\ q & -Q & q' \end{array}) (\begin{array}{ccc} k' & f'' & f' \\ -q' & m'' & m' \end{array}) (\begin{array}{ccc} f'' & K' & f \\ -m'' & Q' & m \end{array}) (\begin{array}{ccc} f & k & f' \\ -m & -q & -m' \end{array}) \times (\begin{array}{ccc} 1 & 1 & K \\ f & f'' & f' \end{array}) \frac{(-1)^{K-Q}}{2K+1} \delta_{K,K'} \delta_{Q,Q'} \{ k' & k & K \\ f & f'' & f' \} \]  

(A.0.31)

to give

\[ \hat{T}^{(K)}_{Q} = (-1)^{K-f-f'} (2f+1)(2f'+1) \alpha_f \times (\begin{array}{ccc} 1 & 1 & K \\ f & f & f' \end{array}) \hat{V}^{(K)}_{Q} \]  

(A.0.32)

Then, with the help of A.0.18 we can express the tensor operators in terms of \( \hat{f} \)
Appendix A: Irreducible representation of the interaction Hamiltonian

operators

\[ \hat{T}_0^{(0)} = -\frac{1}{\sqrt{3}} \hat{\alpha}_f^{(0)} \hat{1}, \]
\[ \hat{T}_Q^{(1)} = \frac{1}{\sqrt{2}} \hat{\alpha}_f^{(1)} \hat{Q}, \]
\[ \hat{T}_0^{(2)} = -\frac{1}{\sqrt{6}} \hat{\alpha}_f^{(2)} (3 \hat{f}_z^2 - \hat{f}^2), \]
\[ \hat{T}_{\pm 1}^{(2)} = -\hat{\alpha}_f^{(2)} (\hat{f}_z \pm \hat{1}/2), \]
\[ \hat{T}_{\pm 2}^{(2)} = -\frac{1}{\sqrt{2}} \hat{\alpha}_f^{(2)} \hat{f}_z^2, \]

where \( \hat{f}_0 = \hat{f}_z \) and

\[ \alpha^{(0)} = (-1)^{2f} \sum_{f'} \hat{\alpha}_f^{(0)} \left( (2f - 1)\delta_{f-1}^{f'} + (2f + 1)\delta_{f}^{f'} + (2f + 3)\delta_{f+1}^{f'} \right) \]
\[ \alpha^{(1)} = (-1)^{2f} \sum_{f'} \hat{\alpha}_f^{(1)} \left( -\frac{2f - 1}{f} \delta_{f-1}^{f'} - \frac{2f + 1}{f(f + 1)} \delta_{f}^{f'} + \frac{2f + 3}{f} \delta_{f+1}^{f'} \right) \]
\[ \alpha^{(2)} = (-1)^{2f} \sum_{f'} \hat{\alpha}_f^{(2)} \left( \frac{1}{f} \delta_{f-1}^{f'} - \frac{2f + 1}{f(f + 1)} \delta_{f}^{f'} + \frac{1}{f} \delta_{f+1}^{f'} \right). \]

Irreducible Hamiltonian

We now bring our attention back to the Hamiltonian A.0.6 which, according to A.0.24, can be written as

\[ \hat{H}_I = \hat{H}_I^{(0)} + \hat{H}_I^{(1)} + \hat{H}_I^{(2)} \]  
(A.0.35)

with

\[ \hat{H}_I^{(K)} = -\sum_{f'} \hat{E}^{(-)} \cdot \hat{\alpha}_f^{(K)} \cdot \hat{E}^{(+)}. \]  
(A.0.36)

By substituting for \( \hat{E}^{(-)} \) and \( \hat{E}^{(+)} \) from A.0.4 and using the definitions of the Stokes vector introduced in Chapter 2, Eq. 2.2.9, we can rewrite the three components of the Hamiltonian (A.0.36) as

\[ \hat{H}_I^{(0)} = \frac{2}{3} g \alpha^{(0)} \hat{S}_0 \hat{1}, \]
\[ \hat{H}_I^{(1)} = g \alpha^{(1)} \hat{S}_f \hat{f}_z, \]  
(A.0.37)
\[ \hat{H}_I^{(2)} = g \alpha^{(2)} \left[ \hat{S}_x (f_y^2 - f_y^2) + \hat{S}_y (f_x f_y + f_y f_x) + \hat{S}_0 (f_z^2 - \hat{f}^2 / 3) \right]. \]
APPENDIX B: AMPTEK A250 SPECIFICATIONS

CHARGE SENSITIVE PREAMPLIFIER

A NEW STATE-OF-THE-ART

- External FET allows matching to detector
- FET can be cooled
- Noise at room temperature \( \equiv 100 \) electrons RMS
- Low power (19 mW typical)

The A250 is a hybrid state-of-the-art Charge Sensitive Preamplifier for use with a wide range of detectors with capacitance from \(< 1\) to several thousand picofarads. Such detectors include silicon, CdTe and Hgl solid state detectors, proportional counters, photomultiplier tubes, piezoelectric devices, photodiodes, CCD's and others.

To permit optimization for a wide range of applications, the input field effect transistor is external to the package and user selectable. This feature is essential in applications where detector and FET must be cooled to reduce noise. In all applications it allows the FET to be matched to the particular detector capacitance, as well as to noise and shaping requirements. In larger quantities, the A250 may be special ordered with internal FET.

The noise performance of the A250 is such that its contribution to FET and detector noise is negligible in all charge amplifier applications, i.e., it is essentially an ideal amplifier in this respect.

The internal feedback components configure the A250 as a charge amplifier; however, it may be used as a high performance current or voltage preamplifier by choice of suitable feedback components.

While these preamps were designed for multidetector satellite instrumentation, their unique characteristics make them equally useful in a broad range of laboratory and commercial applications.

FEATURES
- Ultra low noise
- Low power
- Fast rise time (4ns at Qpf)
- External FET (allows selection or cooling)
- Positive or negative signal processing
- Pin selectable gain
- Small size (14 pin hybrid DIP)
- High reliability screening
- One year warranty

APPLICATIONS
- Aerospace
- Nuclear physics
- Portable instrumentation
- Nuclear monitoring
- Particle, \( \gamma \) and X-ray imaging
- Medical and nuclear electronics
- Electro-optical systems

TYPICAL APPLICATION

AMPTEK INC. 6 DE ANGELO DRIVE, BEDFORD, MA 01730 U.S.A. TEL. (781) 275-2242 FAX. (781) 275-3470 email: sales@amptek.com www.amptek.com

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## Appendix B: AMPTEK A250 Specifications

### Specifications

### Input Characteristics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensitivity (Si)</td>
<td>44 mV/MeV</td>
</tr>
<tr>
<td>(Si)</td>
<td>55 mV/MeV (Ge)</td>
</tr>
<tr>
<td>(Ge)</td>
<td>36 mV/MeV (CdTe)</td>
</tr>
<tr>
<td>(CdTe)</td>
<td>38 mV/MeV (HgI2)</td>
</tr>
<tr>
<td>(HgI2)</td>
<td>1V/μC</td>
</tr>
<tr>
<td>(μC)</td>
<td>0.16 μV/electron</td>
</tr>
</tbody>
</table>

Sensitivity can be reduced by connecting PINS 2 and/or 3 to PIN 1 thus providing C1 = 3, 5, or 7 pF. Additional external capacitance can be added for further reduction of gain. In general, the sensitivity is given by A = 1V/μF V/μC. For Silicon the sensitivity is A = 44C1 / (μF) mV/MeV. Noise: Input FET dependent. See Figure 1. Noise Slope: Input FET dependent. See Figure 1.

Data presented in Figure 1 is representative of results obtained with recommended FETs, and is characteristic of the FET and shaping time constants rather than the A250, which is effectively noiseless. In general, the choice of input FET is based on its noise voltage specification (Vn/H) and its input capacitance (Ciss).

For low capacitance detectors an FET with small Ciss should be chosen, such as 2N4416 or 2SK152.

For very high capacitance detectors, two or more matched high Ciss FETs such as the 2N6550 may be paralleled to achieve the best noise performance.

### Dynamic Input Capacitance

- >40,000 pF with 2X2SK147 FETs and C1 = 5pF

### Polarity

- Negative or Positive

### Output Characteristics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polarity</td>
<td>Inverse of input</td>
</tr>
<tr>
<td>RISE Time</td>
<td>3.8ns at 0.8μF input load with 2SK152</td>
</tr>
<tr>
<td>4.5ns at 100pF input load with 2N6550 or 2SK152</td>
<td></td>
</tr>
<tr>
<td>IMPEDANCE</td>
<td>100 Ohms</td>
</tr>
<tr>
<td>INTEGRAL NONLINEARITY</td>
<td>&lt;0.05% for 0 to +2V unloaded</td>
</tr>
<tr>
<td>DECAY TIME CONSTANT</td>
<td>300 Mohms X C1 = 300μa, 900μa, 1.5ms, 2.1ms</td>
</tr>
<tr>
<td>POSITIVE CLIPPING LEVEL</td>
<td>+2.8V</td>
</tr>
<tr>
<td>NEGATIVE CLIPPING LEVEL</td>
<td>-4.8V</td>
</tr>
<tr>
<td>GENERAL</td>
<td>GAIN-BANDWIDTH PRODUCT</td>
</tr>
<tr>
<td>fr &gt; 300 MHz with 2N4416 FET</td>
<td>—Figure 4.</td>
</tr>
<tr>
<td>fr &gt; 1.5 GHz with two 2SK147 FETs</td>
<td>—Figure 4.</td>
</tr>
</tbody>
</table>

### Operating

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage</td>
<td>±6V, (±8V Maximum)</td>
</tr>
<tr>
<td>Current</td>
<td>±1.2 mA plus the FET drain current (Igs). Where:</td>
</tr>
<tr>
<td>Igs (mA)</td>
<td>30nA</td>
</tr>
<tr>
<td>Igs</td>
<td>2.75 mA</td>
</tr>
<tr>
<td>Power Dissipation</td>
<td>14mW + 6Igs</td>
</tr>
<tr>
<td>Variation of Sensitivity with Supply Voltage</td>
<td>&lt;0.1% from 0 to 100°C</td>
</tr>
<tr>
<td>STABILITY</td>
<td>&lt;0.5% from -55°C to 125°C</td>
</tr>
<tr>
<td>TEMPERATURE</td>
<td>-55°C to +125°C</td>
</tr>
<tr>
<td>STORAGE TEMPERATURE</td>
<td>-65°C to +150°C</td>
</tr>
<tr>
<td>SCREENING</td>
<td>AMPTEK HIGH RELIABILITY</td>
</tr>
</tbody>
</table>

### Package

- 14 PIN Hybrid DIP (Metal)

### Warranty

- One year

### Test Board

- PC-250

### Options

- Internal FET (consult factory)
- NASA GSFC S-311-P-698 screening

### Pin Configuration

<table>
<thead>
<tr>
<th>Pin</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pin 1</td>
<td>300 Mohm resistor in parallel with 1pF feedback capacitor. Connect this pin to the detector and the Gate of the FET.</td>
</tr>
<tr>
<td>Pin 2</td>
<td>2pF feedback tap.</td>
</tr>
<tr>
<td>Pin 3</td>
<td>4pF feedback tap.</td>
</tr>
<tr>
<td>Pin 4</td>
<td>-6V direct.</td>
</tr>
<tr>
<td>Pin 5</td>
<td>-6V through 50 ohm.</td>
</tr>
<tr>
<td>Pin 6</td>
<td>Compensation (0-30pF to ground) for low closed loop gain configuration (where a large feedback capacitor is used together with small detector capacitance).</td>
</tr>
<tr>
<td>Pin 7, 12</td>
<td>Ground and case.</td>
</tr>
<tr>
<td>Pin 8</td>
<td>Output through 100 ohms.</td>
</tr>
<tr>
<td>Pin 9</td>
<td>Output (direct).</td>
</tr>
<tr>
<td>Pin 10</td>
<td>+6V through 50 ohm.</td>
</tr>
<tr>
<td>Pin 11</td>
<td>+6V direct.</td>
</tr>
<tr>
<td>Pin 13</td>
<td>Provide 2.75 mA drain current to the external FET by connecting Pin 13 to 14. (See Operating Current Specifications.)</td>
</tr>
<tr>
<td>Pin 14</td>
<td>Input. Should be connected to the drain of the FET. This pin is held internally at -3 Volts.</td>
</tr>
</tbody>
</table>
Appendix B: AMPTEK A250 Specifications

APPLICATION NOTES

A TWO DETECTOR TELESCOPE SYSTEM

THE A250 CONNECTED TO A SOLID STATE DETECTOR

14 PIN DUAL IN-LINE HYBRID PACKAGE

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Appendix C

Arbitrary magnetic field generator with 50 Hz compensation

Figure C.1: Block diagram.
Figure C.2: Detailed schematic of the circuit.

Appendix C: Arbitrary Magnetic Field Generator with 50 Hz
References


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