Preparation of engineered two-photon entangled states for multidimensional quantum information

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We put forward an experimentally feasible technique to generate engineered entangled states in d-dimensional Hilbert spaces in parametric down-conversion of photons. The scheme is based on the orbital angular momentum of light and translates the classical, topological information imprinted in the light beam that pumps the two-photon source into quantum information contained in the weights and phases of the quantum entangled two-photon states.

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Entanglement of identical particles is one of the most genuine features of the quantum world, and it forms the core of quantum cryptography, computing, and teleportation [1–6]. To date research has focused on quantum states belonging to two-dimensional Hilbert spaces, or qubits. Multidimensional entangled states, or qudits, provide higher dimensional alphabets, thus enhancing the potential of quantum techniques. For example, by using qudits the security of quantum key distribution cryptography can be improved [7], and the efficiency of quantum communication protocols can be enhanced [8]. However, the challenge is the implementation of the d-dimensional quantum channel. Here we put forward a feasible technique to prepare *arbitrary engineered entangled states* in any d-dimensional Hilbert space. The scheme is based on the entanglement of the orbital angular momentum of light [9] in the process of spontaneous parametric down-conversion, and translates topological information imprinted in the pump light into the amplitudes of the generated entangled quantum states. This makes possible the proof-of-principle implementation of capacity-increased protocols based on qudits, and allows the experimental exploration of deeper quantum features, such as hyperentanglement in arbitrary, *on-demand* Hilbert dimensions.

The key ingredient of the scheme put forward here is the engineering of the quantum state by controlling the spatial shape of the laser beam that pumps the down-converter source. The approach bears similarities with the spatial manipulation of pump beams for quantum imaging applications (see, e.g., [10,11]). This is in contrast to the direct manipulation of the quantum state output of the crystal. In particular, manipulation of the orbital angular momentum of the down-converted photons by properly designed holograms modifies the corresponding quantum state allowing, e.g., the generation of states that maximally violate multidimensional versions of Bell inequalities as demonstrated recently by Vaziri, Weihs, and Zeilinger [12]. Such approach relies on the multiple-step manipulation of the *information imprinted on the quantum state*. On the contrary, our scheme relies on the single-step manipulation of the *information imprinted on the classical beam* that pumps the quadratically nonlinear crystal where the desired two-photon quantum state is generated. Thus, it can be readily employed in to generate arbitrary qudits in arbitrary Hilbert dimensions.

Spontaneous parametric down-conversion is a reliable source for the generation of entangled photon pairs [13,14]. Pairs of down-converted photons entangled in polarization, or spin angular momentum, were used, e.g., in the demonstration of quantum teleportation [3], and in the recent realization of a quantum universal NOT gate [4]. However, the down-converted photons can also be entangled in orbital angular momentum (OAM) [9], which belongs to an infinite dimensional Hilbert space and thus allows encoding qudits with arbitrary d [15]. The quantum state of a photon is described by a mode function. Any mode function $\Psi(\rho,\varphi,z)$ with an arbitrary amplitude profile can be expanded into Laguerre Gaussian (LG) modes,

$$\Psi(\rho,\varphi,z)=\sum_{l=-\infty}^{\infty}\sum_{p=0}^{\infty} A_{lp}LG_p^l(\rho,\varphi,z). \tag{1}$$

The normalized Laguerre Gaussian mode at its beam waist $(z=0)$ is given in cylindrical coordinates by

$$LG_p^l(\rho,\varphi)=\sqrt{\frac{2p!}{\pi(|l|+p)!}}\frac{1}{w_0}\left(\frac{\sqrt{2}\rho}{w_0}\right)^{|l|}L_p^{|l|}\left(\frac{2\rho^2}{w_0^2}\right)\times\exp(-\rho^2/w_0^2)\exp(il\varphi), \tag{2}$$

where $L_p^l(\rho)$ are the associated Laguerre polynomials,

$$L_p^{|l|}(\rho)=\sum_{m=0}^{p}(-1)^m\frac{(|l|+p)!}{(p-m)!(|l|+m)!m!}\rho^m, \tag{3}$$

w_0 is the beam width, p is the number of nonaxial radial nodes of the mode, and the index l, referred to as the winding number, describes the helical structure of the wave front around a phase dislocation. When the mode function is a pure LG mode with winding number l, the quantum state is an eigenstate of the OAM operator with eigenvalue $l\hbar$ [16]. State vectors, which are not represented by a pure LG mode, correspond to photons in a superposition state, with the weights of the quantum superposition dictated by the contribution of the lth angular harmonics.

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When a thin quadratic nonlinear crystal of length L is illuminated by a quasimonochromatic laser pump beam propagating in the z direction, with wave number k_p and waist w_0, the generated two-photon quantum state is given by [17]

$$|\Psi\rangle = \int dr_\perp \Phi(r_\perp) a_s^\dagger(r_\perp) a_i^\dagger(r_\perp)|0,0\rangle, \tag{4}$$

where $r_\perp$ is the radial coordinate in the transverse X-Y plane, $|0,0\rangle$ is the vacuum state and $a_s^\dagger$ and $a_i^\dagger$ are creation operators for the signal and idler modes. Here $\Phi(r_\perp)$ is the spatial distribution of the pump beam at the input faced of the crystal. A photon state described by a pure LG mode can be written as

$$|lp\rangle = \int dr_\perp LG_p^l(r_\perp) a^\dagger(r_\perp)|0\rangle. \tag{5}$$

Using $I=\Sigma_{lp}|lp\rangle\langle lp|$, one can express the quantum state $|\Psi\rangle$ using the eigenstates of the orbital angular momentum operator as

$$|\Psi\rangle = \sum_{l_1,p_1}\sum_{l_2,p_2} C_{p_1,p_2}^{l_1,l_2}|l_1,p_1;l_2,p_2\rangle, \tag{6}$$

where (l_1,p_1) correspond to the signal mode and (l_2,p_2) correspond to the idler mode. The expression of the probability amplitude $C_{p_1,p_2}^{l_1,l_2}$ is given by [18,19]

$$C_{p_1,p_2}^{l_1,l_2} \sim \int dr_\perp \Phi(r_\perp)[LG_{p_1}^{l_1}(r_\perp)]^*[LG_{p_2}^{l_2}(r_\perp)]^*. \tag{7}$$

The pump beam $\Phi(r_\perp)$ can be expanded into spiral harmonics to get

$$\Phi(\rho,\varphi) = \sum_{l=-\infty}^{\infty} a_l(\rho)\exp(il\varphi). \tag{8}$$

Therefore, the quantum probability amplitude $C_{p_1,p_2}^{l_1,l_2}$ depends only on the radial profile of the (l_1+l_2)th angular harmonic of the pump beam. Thus, such harmonic content of the pump beam translates to the complex probability amplitude of the quantum states with $l_1+l_2=m$. The weights of the quantum superposition are given by $P_{p_1,p_2}^{l_1,l_2} = |C_{p_1,p_2}^{l_1,l_2}|^2/\eta$, which gives the value of the joint detection probability for finding one photon in the signal mode (l_1,p_1) and one photon in the idler mode (l_2,p_2). The two-photon state produced in the down-conversion process is a coherent superposition of an infinite number of states of the form $|l_1,p_1;l_2,p_2\rangle$, so that using eigenstates with index $l=0,\ldots,d-1$, produces qudits of arbitrary d. We thus consider a detection scheme that projects the OAM two-photon state into d-dimensional Hilbert subspaces, denoted $\mathcal{S}_d$. η is a normalization constant that depends on the particular Hilbert subspace that is considered. For illustrative purposes, here we consider only states with $p_1=p_2=0$, denoted $|l_1,l_2\rangle$.

The technique put forward here is based on the use of a coherent, engineerable superposition of modes as a pump signal. More precisely, we suggest to encode such modes in the form of N single-charge topological screw wave front dislocations, or vortices, nested on a laser beam (e.g., by properly designed computer generated holograms, or reconfigurable spatial light modulators). Such beam can be written as [20]

$$\Phi(\rho,\varphi) = A_0\prod_{j=1}^{N}[\rho\exp(i\varphi)-\rho_j\exp(i\varphi_j)]\exp(-\rho^2/w_0^2), \tag{9}$$

where (ρ_j,φ_j) are the radial and azimuthal positions of the jth vortex, w_0 is the beam width and A_0 is a constant. Projection of the vortex-pancake (9) onto LG modes yields [20]

$$\Phi(\rho,\varphi) = A_0\sqrt{\pi}\sum_{l=0}^{N}(-1)^{N-l}\left(\frac{w_0}{\sqrt{2}}\right)^{l+1} \times\sqrt{\Gamma(l+1)}B_{N-l}LG_0^l(\rho,\varphi), \tag{10}$$

where

$$B_n = \sum_{j_1}\sum_{j_2}\cdots\sum_{j_n}\prod_{l=1}^{n}\rho_{j_l}\exp(i\varphi_{j_l}), \tag{11}$$

with $j_l\in[1,N]$, and $j_l<j_{l+1}$. Using Eqs. (7) and (10) one finds that the spatial field distribution given by Eq. (9) can only generate down-converted entangled photons in a quantum superposition of states with $0\le l_1+l_2\le N$, with

$$a_{l_1+l_2}(\rho) = A_0\sqrt{\pi}(-1)^{N-l_1-l_2}\left(\frac{w_0}{\sqrt{2}}\right)^{l_1+l_2+1}\sqrt{\Gamma(l_1+l_2+1)} \times B_{N-l_1-l_2}u_0^{l_1+l_2}(\rho), \tag{12}$$

where

$$u_0^l(\rho) = LG_p^l(\rho,\varphi)\exp(-il\varphi). \tag{13}$$

This expression is a direct consequence of the conservation of the OAM in spontaneous down-conversion in collinear phase-matching geometries as those considered here, where a pump beam with azimuthal quantum number l_0 can only generate down-converted photons, which yield $l_1+l_2=l_0$. When $p_1=p_2=0$, substitution of Eq. (12) into Eq. (7) yields

$$C_{0,0}^{l_1,l_2} \sim (-1)^{N-l_1-l_2}\frac{2^{(|l_1|+|l_2|)/2}A_0 w_0^{l_1+l_2}}{3^{(l_1+l_2+|l_1|+|l_2|)/2}} \times\frac{\Gamma([l_1+l_2+|l_1|+|l_2|]/2+1)}{\sqrt{\Gamma(|l_1|+1)\Gamma(|l_2|+1)}}B_{N-l_1-l_2}, \tag{14}$$

for $0\le l_1+l_2\le N$. The value of constant B_n depends solely on the location of the vortices. Thus, through the manipulation of the position of the vortices one can control the phase and weight of the probability amplitudes $C_{0,0}^{l_1,l_2}$. This can be

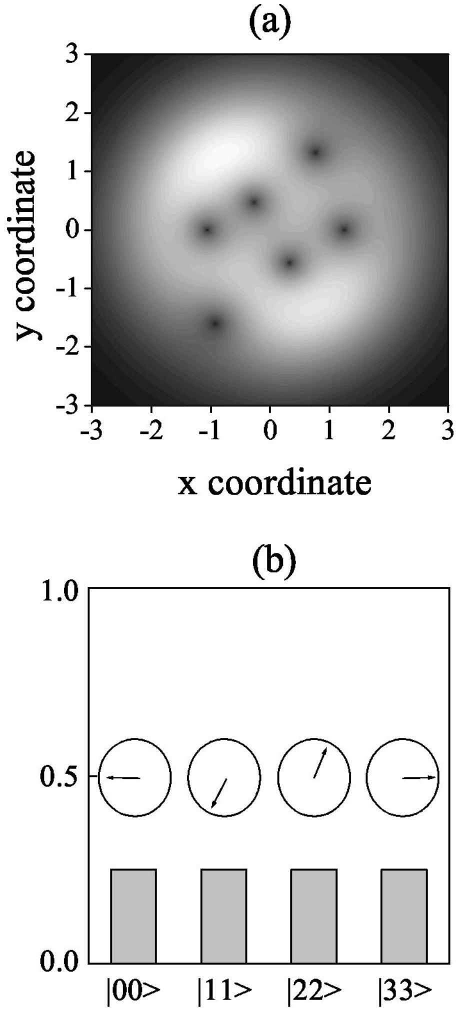

FIG. 1. Intensity profile of the pump beam that generates the maximally entangled qu-quart $\Psi = 1/2[-|00\rangle + \exp(i\theta_1)|11\rangle + \exp(i\theta_2)|22\rangle + |33\rangle]$. (a) Intensity profile, (b) the generated quantum state. In (a), the transverse coordinates are normalized to the beam width w_0. In (b), the bars show the weight distribution and the clocks show the phase distribution of the quantum state. The phases are $\theta_1 = -111°$ and $\theta_2 = 84°$. Locations of the vortex: $\rho_1 = 0.65 w_0$, $\rho_2 = 1.85 w_0$, $\rho_3 = 1.06 w_0$, $\rho_4 = 0.54 w_0$, $\rho_5 = 1.53 w_0$, $\rho_6 = 1.24 w_0$, and $\varphi_i = i\pi/3$ for $i = 1,6$.

employed to generate, e.g., qu-quarts. For example, Fig. 1(a) shows the intensity profile of the pump beam that generates the maximally entangled qu-quart

$$|\Psi\rangle = \tfrac{1}{2}[-|0,0\rangle + \exp(i\theta_1)|1,1\rangle + \exp(i\theta_2)|2,2\rangle + |3,3\rangle], \quad (15)$$

in the subspace $\mathcal{S}_4 = \{|0,0\rangle, |1,1\rangle, |2,2\rangle, |3,3\rangle\}$. The corresponding quantum amplitudes and phases are shown in Fig. 1(b). Additional qu-quarts can be generated by simple topological transformations. For example, a global rotation of the pump by an angle θ generates the new maximally entangled qu-quart

$$|\Psi\rangle = \tfrac{1}{2}[-\exp(i6\theta)|0,0\rangle + \exp(i\theta_1 + i4\theta)|1,1\rangle + \exp(i\theta_2 + i2\theta)|2,2\rangle + |3,3\rangle]. \quad (16)$$

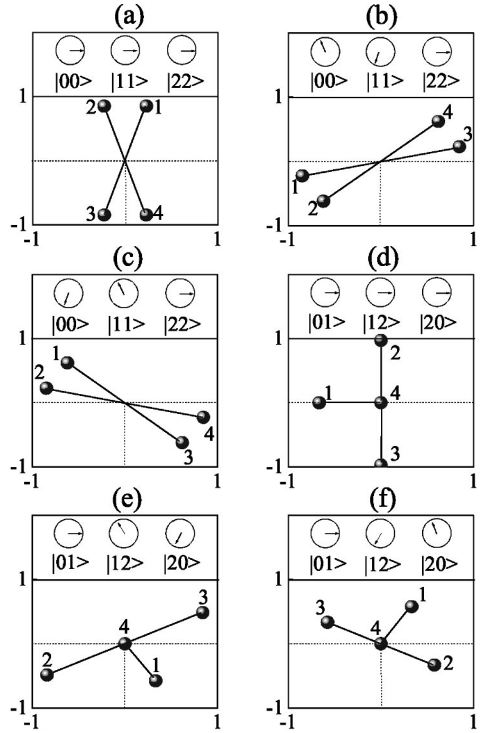

FIG. 2. Diagrams sketching the locations of the topological screw dislocations of the pump beam that generates the six vectors Ψ_{0n} and Ψ_{1n} $(n = 0,1,2)$. The transverse coordinates are normalized to the beam width w_0. The "clocks" show the phase of the amplitude of the corresponding states $|0,0\rangle$, $|1,1\rangle$, and $|2,2\rangle$ for (a)–(c), and for the states $|0,1\rangle$, $|1,2\rangle$, and $|2,0\rangle$ in (d)–(f). (a) $|\Psi_{00}\rangle$, (b) $|\Psi_{01}\rangle$, (c) $|\Psi_{02}\rangle$, (d) $|\Psi_{10}\rangle$, (e) $|\Psi_{11}\rangle$, and (f) $|\Psi_{12}\rangle$. For (a)–(c): $\rho_1 = \rho_2 = \rho_3 = \rho_4 = (16/27)^{1/4}$ and for (d)–(f): $\rho_1 = 2/3$, $\rho_2 = \rho_3 = (\sqrt{8}/3)^{1/2}$, $\rho_4 = 0$. The angles are (a) $\varphi_1 = 75°$, $\varphi_2 = 105°$, $\varphi_3 = -105°$, $\varphi_4 = -75°$, (b) $\varphi_1 = -165°$, $\varphi_2 = -135°$, $\varphi_3 = 15°$, $\varphi_4 = 45°$, (c) $\varphi_1 = 135°$, $\varphi_2 = 165°$, $\varphi_3 = -45°$, $\varphi_4 = -15°$, (d) $\varphi_1 = 180°$, $\varphi_2 = 90°$, $\varphi_3 = -90°$, (e) $\varphi_1 = -60°$, $\varphi_2 = -150°$, $\varphi_3 = 30°$, and (f) $\varphi_1 = 60°$, $\varphi_2 = -30°$, $\varphi_3 = 150°$.

There are d^2 maximally entangled states that form an orthonormal base for the space of two entangled qubits. The explicit form of one base is

$$|\Psi_{m,n}\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} \exp(i2\pi jn/d)|j,(j+m)_{\text{mod-}d}\rangle, \quad (17)$$

where m and n run from 0 to $d-1$. For $d = 2$, they correspond to the usual Bell states for qubits. The corresponding base, but for qutrits, can be generated by a pump beam with $N = 4$, which corresponds to four vortices nested off axis. The diagrams of Fig. 2 sketch the location of the vortices needed to generate the six vectors corresponding to the bases $|\Psi_{0n}\rangle$ and $|\Psi_{1n}\rangle$ with $n = 0,1,2$. The maximally entangled states $|\Psi_{0n}\rangle$ describe a base of the subspace $\mathcal{S}_3 = \{|0,0\rangle, |1,1\rangle, |2,2\rangle\}$, while $|\Psi_{1n}\rangle$ is the corresponding base for $\mathcal{S}_3 = \{|0,1\rangle, |1,2\rangle, |2,0\rangle\}$. The probability amplitudes of states $|l_1, l_2\rangle$ and $|l_2, l_1\rangle$ are always found to be equal, thus the same topology of the pump beam can generate the quantum states $|\Psi_{1n}\rangle$ and $|\Psi_{2n}\rangle$. Figures 3(a)–3(c) show the

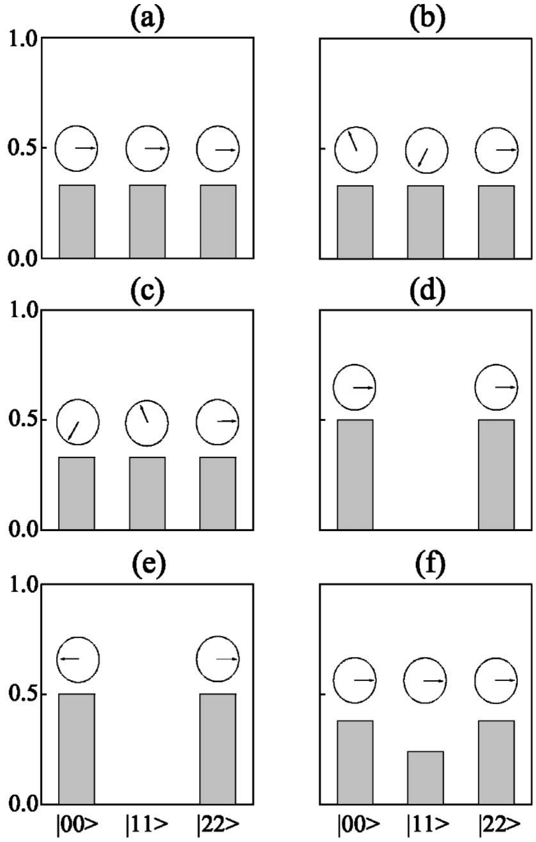

FIG. 3. Generation of selected quantum superposition states. (a) $|\Psi_{00}\rangle$, (b) $|\Psi_{01}\rangle$, (c) $|\Psi_{02}\rangle$, (d) $|\Psi\rangle=1/\sqrt{3}[|0,0\rangle+|2,2\rangle]$, (e) $|\Psi\rangle=1/\sqrt{3}[-|0,0\rangle+|2,2\rangle]$, and (f) $|\Psi\rangle=1/\sqrt{2+\gamma^2}[|0,0\rangle+\gamma|1,2\rangle+|2,2\rangle]$ with $\gamma=(\sqrt{11}-\sqrt{3})/2$. The bars show the weight of the projections into pure modes and the clocks show the phases of the quantum states.

weights and quantum phases of the three states $|\Psi_{0n}\rangle$, all of which are maximally entangled.

We next illustrate with two examples how the scheme can be employed to generate engineered quantum states for optimal quantum protocols. Consider a pump topology with four off-axis vortices equiseparated from the beam axis, at a distance $\rho_1=\rho_2=\rho_3=\rho_4=(16/27)^{1/4}$, and with angular locations given by $\varphi_1=\theta+\Delta$, $\varphi_2=\theta-\Delta+\pi$, $\varphi_3=\theta+\Delta+\pi$, and $\varphi_4=\theta-\Delta$. Such topology generates quantum entangled states belonging to the subspace $\mathcal{S}_3=\{|0,0\rangle,|1,1\rangle,|2,2\rangle\}$, with the general form

$$|\Psi\rangle=\frac{1}{\sqrt{2+4\cos^2(2\Delta)/3}}\bigg[\exp(i4\theta)|0,0\rangle - \frac{2}{\sqrt{3}}\cos(2\Delta)\exp(i2\theta)|1,1\rangle+|2,2\rangle\bigg]. \tag{18}$$

Different possibilities are possible. For example, when $\Delta=75°$ and $\theta=0°,60°,120°$, the quantum states generated correspond to the three vectors $|\Psi_{0n}\rangle$, respectively, which are maximally entangled states with equal amplitudes but different phases of the constituent pure states. When $\Delta=15°$, the state is still a maximally entangled state, but now with a quantum phase reversal imposed to the state $|1,1\rangle$. When $\Delta=45°$, the contribution of the state $|1,1\rangle$ is suppressed, and by rotating the whole configuration by θ, one can generate quantum states with an arbitrary phase delay between $|0,0\rangle$ and $|2,2\rangle$. Two important concrete examples are shown in Figs. 3(d) and 3(e).

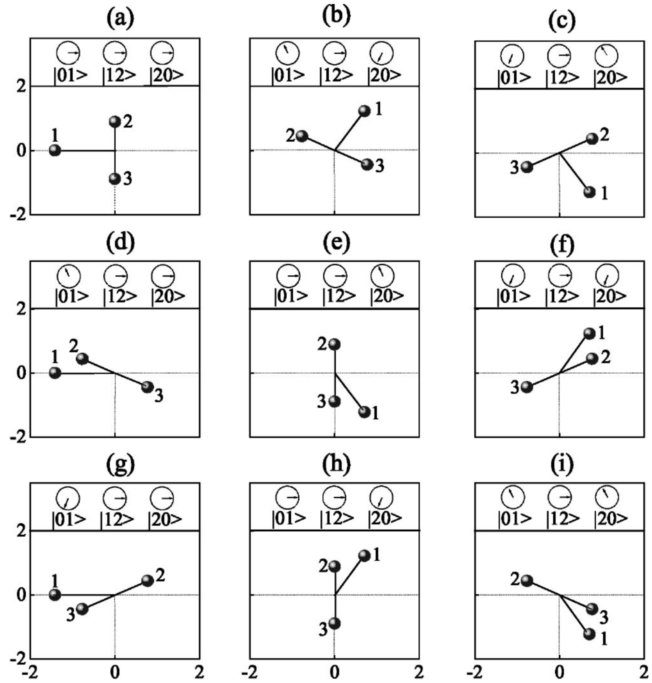

FIG. 4. Generation of nine quantum vectors for three mutually unbiased bases. The transverse coordinates are normalized to the beam width w_0. The clocks show the phase of the amplitude of the corresponding states $|0,1\rangle$, $|1,2\rangle$, and $|2,0\rangle$. In all cases $\rho_1=\sqrt{2}$, $\rho_2=\rho_3=[2\sqrt{2}/3]^{1/2}$, and $\varphi_3=\varphi_2+\pi$. (a) $\varphi_1=180°$, $\varphi_2=90°$, (b) $\varphi_1=60°$, $\varphi_2=150°$, (c) $\varphi_1=-60°$, $\varphi_2=30°$, (d) $\varphi_1=180°$, $\varphi_2=150°$, (e) $\varphi_1=-60°$, $\varphi_2=90°$, (f) $\varphi_1=60°$, $\varphi_2=30°$, (g) $\varphi_1=180°$, $\varphi_2=30°$, (h) $\varphi_1=60°$, $\varphi_2=90°$, and (i) $\varphi_1=-60°$, $\varphi_2=150°$.

Direct applications of the possibilities open by Eq. (18) occur in all d-dimensional quantum communication protocols. For example, Brukner and co-workers showed that the efficiency of quantum two-party communication complexity problems can be enhanced over any classical protocol by using qutrits [8], provided that they violate Bell inequalities in the form defined by Collins *et al.* [21], and Acin *et al.* showed that the qutrit

$$|\Psi\rangle=\frac{1}{\sqrt{2+\gamma^2}}|0,0\rangle+\gamma|1,1\rangle+|2,2\rangle, \tag{19}$$

with $\gamma=(\sqrt{11}-\sqrt{3})/2$, violates Bell inequality stronger than any maximally entangled state [22]. Such state can be readily generated with our technique: The topology required to generate such state shown in Fig. 3(f) is $\theta=0$ and $\cos(2\Delta)=-\sqrt{3}\gamma/2$ (i.e., $\Delta\simeq67°$).

Another direct application of the technique holds in generalizations of the Bennet-Brassard quantum key distribution protocol [24], which make use of N-dimensional systems and M mutually unbiased bases. To set a secret key requires that Alice chooses one of the $M\times N$ vectors and sends a quantum state represented by that vector. The use of higher values of N and M provides better security than obtainable with qubits and two bases [7,23]. One can construct $M=d+1$ mutually unbiased bases with $d=p^k$, where p is a prime number and k an integer [25]. This can also be implemented with our technique using the set $\mathcal{S}_3=\{|0,1\rangle,|1,2\rangle,|2,0\rangle\}$ and a $N=3$ vortex pancake with $\rho_1=\sqrt{2}$, $\rho_2=\rho_3=[2\sqrt{2}/3]^{1/2}$ and $\varphi_3=\varphi_2+\pi$. By tuning φ_1 and φ_2, one can generate the nine vectors of the form $|\Psi\rangle=1/\sqrt{3}[\mu|0,1\rangle+\nu|2,0\rangle+|1,2\rangle]$, where μ and ν are given by any combination $\mu,\nu=1,\exp(i2\pi/3),\exp(-i2\pi/3)$, which allow to generate three of the mutually unbiased bases. An additional base can be obtained by generating the states $|0,1\rangle$, $|2,0\rangle$, and $|1,2\rangle$, by using pump beams with a $l=1,2,3$ on-axis vortex (Fig. 4).

We conclude by noticing that pancakes of topological dislocations can be nested in host laser beams by different methods, including computer-generated holography and spatial light modulators [26–29]. Holographic mode splitters similar to those recently employed to demonstrate the violation of Bell inequalities with qutrits [12] can be used to project the quantum states into the Hilbert subspaces. Such experimental demonstration confirms that the scheme proposed here can be implemented within the current technological state of the art in the manipulation and detection of OAM eigenstates. We finally conjecture that OAM topologies should be storable in quantum memories made of cold atoms [30,31], and phase imprinted in atomic-molecular Bose-Einstein condensates [32], for the matter implementation of the technique.

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