Arresting Wave Collapse by Wave Self-Rectification

Lucian-Cornel Crasovan,* Juan P. Torres, Dumitru Mihalache,* and Lluis Torner
ICFO-Institut de Ciencies Fotoniques, and Department of Signal Theory and Communications, Universitat Politecnica de Catalunya, 08034 Barcelona, Spain
(Received 12 March 2003; published 8 August 2003)

We put forward a mechanism for tailoring, and even arresting, the collapse of wave packets in nonlinear media, whose dynamics is governed by nonlocal two-dimensional nonlinear Schrödinger-like equations. The key ingredient of the scheme is the self-generation of nonlocal nonlinearities mediated by wave rectification.

DOI: 10.1103/PhysRevLett.91.063904 PACS numbers: 42.65.Jx, 42.65.Sf, 42.65.Tg, 78.20.Jq

The exploding interest taken by the study of the nonlinear Schrödinger equation (NLSE) in the past decades highlights its general features and its applicability in almost all branches of physics [1]. The $(1 + 1)$-dimensional version of the NLSE is an integrable model and possesses both single soliton and multisoliton solutions. On the contrary, the higher-dimensional NLSE, i.e., the $(2 + 1)$-dimensional and the $(3 + 1)$-dimensional models, are no longer integrable. However, they possess stationary solutions, which are unstable on propagation. Maybe the most fascinating issue related to the higher-dimensional NLSE is that, for a wide range of initial conditions, the system evolution shows collapse [2]. Collapse was theoretically predicted for the $(2 + 1)$-dimensional NLSE back in the 1960s [3]. Although close to the point of collapse, the NLSE fails to describe correctly the evolution since some of the assumptions made in its derivation are violated [4], the initial stages of the propagation keep signatures of the collapsing effect that can be experimentally observed. In particular, filamentation of light beams was observed indeed in the early days of nonlinear optics [5], and important mathematical and numerical effort was developed to understand the origin of this effect in both $(2 + 1)$- and $(3 + 1)$-dimensional versions of NLSE [6–8]. Other physical systems described by NLSE-like models, such as Bose-Einstein condensates, might suffer collapse also. An indirect evidence of such a phenomenon is the appearance of a violent burst of cold atoms, an effect which was thus termed *Bosenova*, that was observed in ^{85}Rb condensates when the interaction between atoms is attractive [9,10].

Therefore, an important challenge in nonlinear science is to find out mechanisms arresting the collapse in NLSE models. In more general models which include higher order effects as well as nonparaxial corrections, it has been shown that, under specific conditions, the collapse is arrested [11–13]. Recently it was shown numerically that the use of structures with periodic nonlinearities (or tandems [14]), consisting of self-focusing and defocusing layers can arrest collapse and lead to $(2 + 1)$-dimensional quasistationary propagation [15]. Such a mechanism has been suggested recently to arrest the collapse in Bose-Einstein condensates when the interactions periodically vary between repulsive and attractive by using fast-oscillating Feshbach resonances [16]. Another potential approach is the use of nonlocal nonlinearities. In nematic liquid crystals, nonlocal cubic nonlinearities can lead to the formation of spatial solitons [17], whereas in Bose-Einstein condensates specific types of nonlocal attractive interactions were shown to arrest the collapse [18].

Quadratic nonlinearities are known to be collapse-free, a feature based on the existence of families of stable solitons in both two-dimensional and three-dimensional geometries in nearly phase-matched frequency-conversion processes, including the case of competing quadratic and cubic nonlinearities [19,20]. However, light beams in quadratic nonlinear media interact also with themselves via cascaded optical rectification (OR) [the polarization at the zero frequency is produced by $P(\omega = 0) \sim \chi^{(2)}(0, -\omega, \omega)A^*A$] and the electro-optic (EO) effect. Such self-action can be employed to induce nonlocal nonlinearities, a possibility which opens the door to a new mechanism for the control of light waves [21–25]. In this Letter, we show that under appropriate conditions such a mechanism greatly enhances the threshold intensity that yields collapse induced by self-focusing cubic nonlinearities.

Let the coordinate axes (x, y, z) coincide with the crystal optical axes (a, b, c) of a noncentrosymmetric crystal with quadratic and cubic nonlinearities, so that $\{x \parallel c, y \parallel a, z \parallel b\}$, under conditions of largely mismatched frequency-conversion processes. The direction of light propagation is z and the beam is linearly polarized along x. Under such circumstances, for materials with point group symmetry $mm2$, such as potassium titanyl phosphate (KTP) and potassium niobate ($KNbO_3$), the normalized evolution equations in the slowly varying envelope approximation are written [25]

$$i\frac{\partial U}{\partial z} + \frac{1}{2}\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right) + |U|^2 U - \rho V U = 0, \quad (1)$$

$$\frac{\partial^2 V}{\partial x^2} + \nu\frac{\partial^2 V}{\partial y^2} = \frac{\partial^2 |U|^2}{\partial x^2}, \quad (2)$$

where U is the normalized amplitude of the envelope of the optical beam A_x ($U = A_x/A_0$) and V is the normalized rectified, or static, field E_x ($V = E_x/E_0$). Materials with other point group symmetries fulfill equations similar to Eqs. (1) and (2). The transverse coordinates x and y are normalized to the beam width η, and z is the longitudinal coordinate normalized to twice the diffraction length, which is written $L_d = k_0 n_3 \eta^2/2$. Here k_0 is the wave number in vacuum and n_3 is the refractive index at frequency ω for x-polarized beams. If one makes use of $\alpha = (n_3^8 r_{333}^2/4)/(\epsilon_{33} + 2)$, $\mu = n_3^4 r_{333}$, and $\gamma = 3/4\,\chi^{(3)}_{3333}$, where r_{333} is the electro-optic coefficient, $\chi^{(3)}_{3333}$ is the third order nonlinear coefficient, and ϵ_{33} is the static dielectric constant, one finds that the fields A_0 and E_0 are given by

$$A_0^2 = \frac{\lambda n_3}{2\pi L_d(\gamma + \alpha)}, \qquad E_0 = \frac{\mu}{4\epsilon_{33}} \frac{\lambda n_3}{2\pi L_d(\gamma + \alpha)}, \quad (3)$$

where λ is the wavelength of the radiation. Evolution equations (1) and (2) are characterized by two parameters: the coupling constant $\rho = \mu^2/[4\epsilon_{33}(\gamma + \alpha)]$, which comes from the combined OR and EO effects, and the asymmetry parameter $\nu = \epsilon_{11}/\epsilon_{33}$, which comes from the anisotropy of the material. In order to grasp the value of the different parameters involved, let us consider a laser beam at $\lambda \simeq 1\ \mu$m propagating in $KNbO_3$. One has $n_3 = 2.1194$, $\epsilon_{11} = 37$, $\epsilon_{33} = 24$, $r_{333} = 30.5$ pm/V, and $\chi^{(3)}_{3333} \simeq 45 \times 10^{-22}$ m^2/V^2 [22]. For a beamwidth $\eta \simeq 20\ \mu$m, we have $L_d = 2.5$ mm, $\rho \simeq 0.5$, and $\nu \simeq 1.5$. Then $|U|^2 \simeq 1$ corresponds to an intensity of $\simeq 5$ GW/cm^2 and $|V| \simeq 1$ corresponds to $\simeq 1$ kV/cm.

Equations (1) and (2) are of a Davey-Stewartson (DS) type [26], and arise in different physical settings. DS equations can support solutions that collapse at a finite distance [27] and, for certain coefficients, they are completely integrable. Under general conditions in a quadratic crystal, integrability corresponds to fixed relations between the material parameters [28]. Equations (1) and (2) would be integrable when $\rho = 2$, $\nu = -1$, conditions that in our case cannot be fulfilled ($\nu > 0$).

When the nonlinear polarization induced by the combined OR and EO effects ($\rho = 0$) is neglected, Eqs. (1) and (2) decouple, thus Eq. (1) becomes the (2 + 1)-dimensional NLSE. Its stationary solutions have a constant power $I = 5.85$ [29], where $I = \iint |U|^2 dx\, dy$. For a given initial beam profile $U(x, y, z = 0)$, the beam evolution described by the (2 + 1)-dimensional NLSE either shows diffraction if the power is below a critical power, or it shows blowup in a finite distance if the power is higher than the critical value. The power of the stationary solution is a lower bound for the critical power for blowup [12]. When $\rho \neq 0$, Eqs. (1) and (2) are coupled and the refractive index change due to the presence of a static field is given in normalized units by $\Delta n = |U|^2 - \rho V$. Contrary to the case of the NLSE, the nonlinearity due to OR and EO effects is nonlocal, since the refractive index change Δn at the position (x, y) depends on the shape of the whole beam through Eq. (2). Since the effect of the rectified field is to modify the nonlinearly induced refractive index change [25,30], we expect that the coupling of rectified and optical fields can drastically affect the evolution dynamics for blowup. To elucidate the actual wave evolution, we solved numerically the coupled equations (1) and (2). At each propagation step z, we first calculate the rectified field V for a given optical beam profile $U(x, y, z)$, by solving Eq. (2). Thereafter we solve Eq. (1) for the optical field with a standard split-step beam propagation method algorithm whereas the static field is obtained from Eq. (2): $V(x, y) = \mathcal{F}^{-1}\{k_x^2/(k_x^2 + \nu k_y^2)\mathcal{F}(|U|^2)\}$; $\mathcal{F}$ and $\mathcal{F}^{-1}$ being the direct and inverse Fourier transforms and $k_{x,y}$ the Fourier frequencies in directions x and y.

For the sake of simplicity, we discuss here only the evolution of beams with an initial Gaussian beam profile $U(x, y, z = 0) = [2I/(\pi w_x w_y)]^{1/2} \exp(-x^2/w_x^2 - y^2/w_y^2)$. Notwithstanding, we have made simulations with other kinds of initial beam profiles and we have verified that our main conclusions hold for more general input beams. Let us consider a symmetric input beam ($w_x = w_y = 1$) with power $I = 10$, which is above the critical power for blowup of input Gaussian beams whose evolution is described by the (2 + 1)-dimensional NLSE. Figures 1(a) and 1(b) show the evolution of the peak amplitude of the beam for different values of the coupling constant ρ when the asymmetry parameter ν is kept fixed. The curves for $\rho = 0$ correspond to the evolution described by the (2 + 1)-dimensional NLSE. In all cases, we notice that, when the defocusing effect due to the coupling between the rectified and optical fields is strong enough, by increasing the parameter ρ, the blowup of the beam can be arrested, and this turns out to be the case for all the values of ν considered. For a given value of ν, there is a critical value of ρ, above which one has collapse-free propagation. For small values of ρ, the blowup, although still present, is slowed down when compared to the case when the effect of the rectified field is neglected ($\rho = 0$). Thus, for a given input power, there exists a border in the parameter plane (ρ, ν) that separates the collapsing and the no-collapsing regimes for symmetric Gaussian input beams [see Fig. 1(c)]. The effect of ν is to modify the rectified field generated for a given optical beam shape, which is a direct result of the nonlocal nature of the nonlinearity induced by the combined OR and EO effects. For completeness, we have also determined the threshold collapse power $I_{\rm th}$ as a function of ρ for a fixed value of the asymmetry parameter $\nu = 1.5$. Notice that at large values of ρ, e.g., $\rho = 2$, the threshold intensity is more than 3 times larger than the corresponding NLSE critical power [see Fig. 1(d)]. We have also simulated the cases when the initial power is below the critical power for collapse ($I < 5.85$). In this case, the presence of the rectified electric field enhances the diffraction of the beam. In all cases investigated here, we always found either collapse or diffraction, with no oscillatory evolution towards a

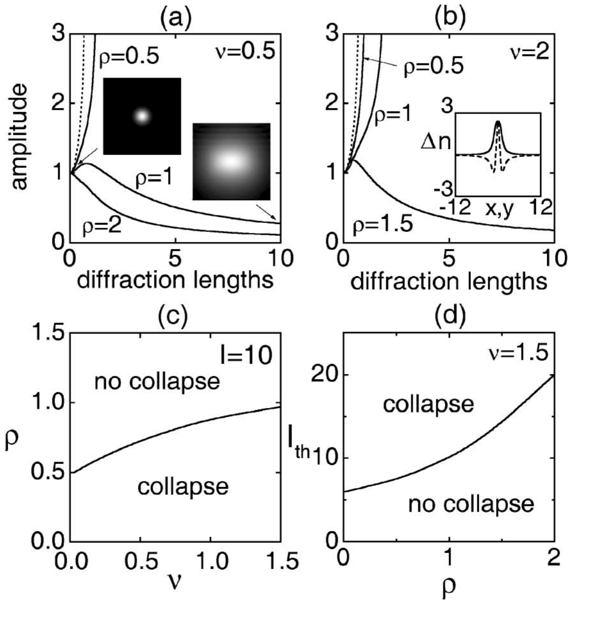

FIG. 1. (a),(b) Evolution of the peak amplitude (normalized to the peak amplitude of the input beam) of the optical beam for two representative values of ν. Dotted line: evolution described by the (2 + 1)-dimensional NLSE ($\rho = 0$). The insets in (a) show the optical intensity beam profile at $z = 0$ and $z = 5$ for $\rho = 1$, whereas the inset in (b) shows the normalized refractive index change Δn along the x and y axes for $\rho = 1.5$. Solid and dashed lines: refractive index change along the x and y axes, respectively. (c) Border between the collapse and the no-collapse regions in the (ρ, ν) plane. (d) Threshold collapse power as a function of ρ for a fixed ν. In all cases, Gaussian beams with $w_x = w_y = 1$ were considered and in panels (a)–(c) we set $I = 10$.

stationary state, an indication that the system does not possess attractor stable solutions. Moreover, in most of the cases, we have seen an enhancement of the peak amplitude of the field in the first stages of propagation, a feature also observed for the elliptic Gaussian input beams [31].

One important feature observed in the simulations is that the initially symmetric beam turns into an elliptical one, as shown in the insets of Fig. 1(a) for $\nu = 0.5$ and $\rho = 1$. Equation (2) is equivalent to the equation that describes the static electric field distribution generated by an equivalent normalized charge density source $\partial^2|U|^2/\partial x^2$, which does not hold the symmetry of the optical field. Therefore, the induced refractive index change is not symmetric although the optical field might be. In the inset of Fig. 1(b), we show the typical normalized refractive index change at $z = 0$ for a noncollapsing regime ($\rho = 1.5$), along both x and y axes. Similar features are observed in a collapsing regime ($\rho = 0.5$). Notice that the refractive index change curve is narrower along the y direction, so that focusing is stronger along the y direction than along the x direction, which explains the stronger focusing effect along the y direction, the direction perpendicular to the polarization of the optical beam.

The effect of the nonlinearity (i.e., the refractive index change self-induced) depends on the shape of the whole beam, so the evolution of the optical beam can be tailored by controlling the ellipticity (e) of the input beam [31], which we define as $e = \log(w_x/w_y)$. We have investigated the evolution of nonsymmetric, i.e., elliptical input beams. We take $\rho = 0.5$ and $\nu = 1.5$, typical values corresponding to the propagation of focused beams in $KNbO_3$. Figure 2(a) shows comparatively the evolution of an elliptical beam for several values of the input beam ellipticity. The power is kept fixed ($I = 10$) in all cases. The presence of the rectified field cannot arrest blowup of the initially circular beam. For an initially highly elliptical beam, the blowup is arrested and the beam transforms to a nearly circular beam during evolution, as shown in the panels of Fig. 2(c) for a beam with $e = -0.477$ (w_x:w_y = 1:3). In order to gain a deeper insight into the effect of the presence of the rectified field, we show in Fig. 2(b) the normalized refractive index change induced (Δn) at the center of the beam as a function of the ellipticity of the input beam. The main conclusion from Fig. 2(b) is that for highly elliptical input beams, focused along the direction (x) parallel to the polarization of the optical field, the combined OR and EO effects reduce the amount of refractive index change induced by the nonlinearity, therefore the blowup of the beam can be arrested [22]. This dependence of the nonlinear effects on the shape of the optical beam has been proposed as a mechanism to induce tunable competing nonlinearities [32].

Panels (a) and (b) of Fig. 3 show the peak amplitude and the beamwidth in the x and y directions of the beam after four diffraction lengths ($z = 2$) for different values of the ellipticity of the input beam for the case with $\nu = 1.5$ and $\rho = 0.5$. As the input ellipticity grows, Fig. 3 shows that the peak amplitude of the output beam

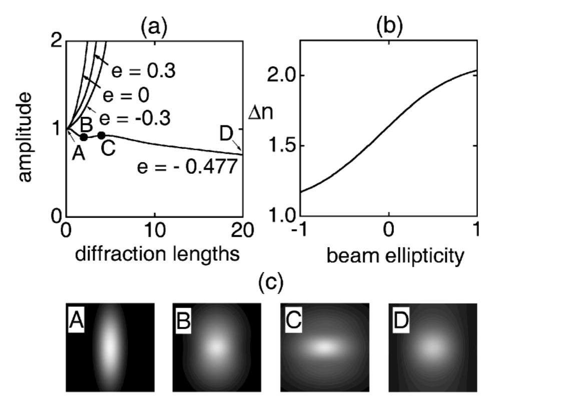

FIG. 2. (a) Evolution of the peak amplitude of the optical beam (normalized to the peak amplitude of the input beam) for different ellipticities. (b) Normalized refractive index change at the center of the beam at $z = 0$ for several input beams with different beam ellipticities. (c) Snapshots of the beam profile corresponding to the points A through D in Fig. 3(a). Beam parameters: $\nu = 1.5$, $\rho = 0.5$, $I = 10$.

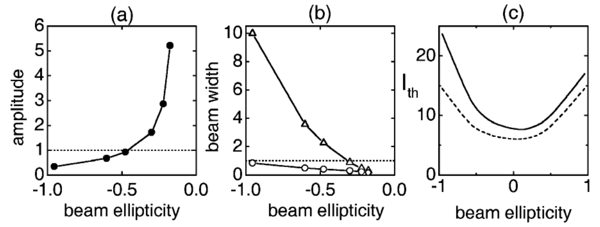

FIG. 3. Peak amplitude (a) and beamwidth in the x and y directions (b) (normalized to their input values) at $z = 2$, versus beam ellipticity. Open triangles: width in the x direction; open circles: width in the y direction. (c) Critical collapse power versus beam ellipticity. Dashed line: $(2 + 1)$-dimensional NLSE case. Parameters: $\nu = 1.5$, $\rho = 0.5$, $I = 10$.

decreases while at the same time the beam width increases, which is a signature of the transition of the regime where the collapsing effect dominates for nearly symmetric beams to the regime where propagation is collapse-free and diffraction dominates for highly elliptical beams. In panel (c) of Fig. 3, we plot the threshold collapse power for Gaussian-shaped input beams versus the beam ellipticity for the same crystal parameters as above. The comparison with the NLSE case [the dashed curve in this panel, calculated as per Eq. (9) in Ref. [31]] shows that the threshold collapse power can be made considerably larger (up to 40%) by means of competing EO and OR effects. However, for crystals with larger coupling parameters ρ, this collapse threshold can be much higher [see Fig. 1(d)].

In conclusion, we have shown that the interaction of beams with self-generated static fields can drastically impact the beam dynamical evolution. In particular, we found that the refractive index change induced via combined optical rectification and electro-optic effects mediated by quadratic nonlinearities can even suppress the otherwise violent collapse induced by the intrinsic cubic nonlinearities. The process described holds directly for light beams but we advance that analogous effects might also occur in atomic-molecular Bose-Einstein condensates.

Support from Generalitat de Catalunya, Spanish Government/BFM2002-2861, and ICREA (D. M.) is aknowledged.

*Also with Department of Theoretical Physics, Institute of Atomic Physics, P.O. Box MG-6, Bucharest, Romania.

[1] A. C. Newell and J. V. Moloney, *Nonlinear Optics* (Addison-Wesley, Redwood City, 1991); N. N. Akhmediev and A. Ankiewicz, *Solitons: Nonlinear Pulses and Beams* (Chapman & Hall, London, 1997).
[2] For a comprehensive review of wave collapse in physics, see L. Bergé, Phys. Rep. **303**, 260 (1998), and references therein.
[3] P. L. Kelley, Phys. Rev. Lett. **15**, 1005 (1965).
[4] M. D. Feit and J. A. Fleck, J. Opt. Soc. Am. B **5**, 633 (1988).
[5] Y. R. Shen, Rev. Mod. Phys. **48**, 1 (1976).
[6] V. E. Zakharov and A B. Shabat, Sov. Phys. JETP **34**, 62 (1972); V. E. Zakharov and V. S. Synakh, Sov. Phys. JETP **41**, 465 (1976).
[7] K. Rypdal, J. J. Rasmussen, and K. Thomsen, Physica (Amsterdam) **16D**, 339 (1985).
[8] E. A. Kuznetsov *et al.*, Physica (Amsterdam) **87D**, 273 (1995).
[9] E. A. Donley *et al.*, Nature (London) **412**, 295 (2001).
[10] J. L. Roberts *et al.*, Phys. Rev. Lett. **86**, 4211 (2001).
[11] J. M. Soto-Crespo and N. Akhmediev, Opt. Commun. **101**, 223 (1993); N. Akhmediev, A. Ankiewicz, and J. M. Soto-Crespo, Opt. Lett. **18**, 411 (1993).
[12] G. Fibich, Phys. Rev. Lett. **76**, 4356 (1996); G. Fibich and A. L. Gaeta, Opt. Lett. **25**, 335 (2000).
[13] P. M. Goorjian and Y. Silberberg, J. Opt. Soc. Am. B **14**, 3253 (1997).
[14] L. Torner *et al.*, IEEE Photon. Technol. Lett. **11**, 1268 (1999); Opt. Commun. **199**, 277 (2001).
[15] I. Towers and B. A. Malomed, J. Opt. Soc. Am. B **19**, 537 (2002).
[16] F. Kh. Abdullaev *et al.*, Phys. Rev. A **67**, 013605 (2003); H. Saito and M. Ueda, Phys. Rev. Lett. **90**, 040403 (2003).
[17] M. Peccianti, K. A. Brzdakiewicz, and G. Assanto, Opt. Lett. **27**, 1460 (2002).
[18] V. M. Pérez-García, V. V. Konotop, and J. J. García-Ripoll, Phys. Rev. E **62**, 4300 (2000).
[19] G. I. Stegeman *et al.*, Opt. Quantum Electron. **28**, 1691 (1996); A. V. Buryak *et al.*, Phys. Rep. **370**, 63 (2002).
[20] A. V. Buryak, Y. S. Kivshar, and S. Trillo, Opt. Lett. **20**, 1961 (1995); O. Bang, J. Opt. Soc. Am. B **14**, 51 (1997); L. Bergé *et al.*, Phys. Rev. E **55**, 3555 (1998); O. Bang *et al.*, Phys. Rev. E **58**, 5057 (1998).
[21] T. K. Gustafson *et al.*, Opt. Commun. **2**, 17 (1970); Ch. Bosshard *et al.*, Phys. Rev. Lett. **74**, 2816 (1995).
[22] I. Biaggio, Phys. Rev. Lett. **82**, 193 (1999); Phys. Rev. A **64**, 063813 (2001).
[23] Ch. Bosshard *et al.*, Opt. Lett. **24**, 196 (1999).
[24] M. J. Ablowitz, G. Biondini, and S. Blair, Phys. Lett. A **236**, 520 (1997); Phys. Rev. E **63**, 046605 (2001); H. Leblond, J. Phys. A **31**, 3041 (1998).
[25] J. P. Torres *et al.*, Opt. Commun. **213**, 351 (2002).
[26] A. Davey and K. Stewartson, Proc. R. Soc. London A **338**, 101 (1974); see also M. J. Ablowitz and H. Segur, *Solitons and the Inverse Scattering Transform* (SIAM, Philadelphia, 1981), and references therein.
[27] G. C. Papanicolaou *et al.*, Physica (Amsterdam) **72D**, 61 (1994); G. Fibich and G. Papanicolau, SIAM J. Appl. Math. **60**, 183 (1999).
[28] H. Leblond, J. Phys. A **31**, 5129 (1998).
[29] Y. Silberberg, Opt. Lett. **15**, 1282 (1990).
[30] R. Spreiter, C. Bosshard, and P. Gunter, J. Opt. Soc. Am. B **18**, 1311 (2001).
[31] G. Fibich and B. Ilan, J. Opt. Soc. Am. B **17**, 1749 (2000).
[32] J. P. Torres *et al.*, Opt. Lett. **27**, 1631 (2002).