Eigenvalue control and switching by fission of multisoliton bound states in planar waveguides

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We report the results of numerical studies of the fission of N-soliton bound states at the interface formed by a Kerr nonlinear medium and a linear dielectric in a planar waveguide. A variety of effects are shown to occur, with applications to all-optical eigenvalue soliton control. © 2004 Optical Society of America

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Bound states (BSs) of N solitons exist in Kerr nonlinear media in one-dimensional geometries (i.e., optical fibers or planar waveguides) and are formed when the amplitude of the input signal is N times higher than the amplitude corresponding to the exact fundamental soliton. Within the framework of the inverse-scattering transform, BSs are nonlinear superpositions of N solitons with amplitudes from 1 to $2N - 1$ and equal velocities (temporal solitons) or propagation angles (spatial solitons). The binding energy for BSs in Kerr media, described by the integrable nonlinear Schrödinger equation, is zero. Therefore under the influence of perturbations the BSs can decay into a set of fundamental solitons hidden in the input profile. The breakup of temporal BSs in optical fibers has been extensively studied.[1–9] In particular, BS breakup by induced fission is a key ingredient in the eigenvalue communications scheme introduced in Ref. 10 for security-enhanced data transmission in telecommunication links. Controllable BS fission is also at the core of the related concept of eigenvalue switching proposed for spatial solitons.[11]

In this context, the breakup of spatial BSs induced by the reflection at material interfaces is particularly interesting. The interactions and switching of single beams,[12–16] soliton pairs,[17] and vector solitons[18] with a nonlinear interface have already been studied. However, although the interaction of a BS with a nonlinear interface is known to lead to splitting of the BS,[19,20] the implication of the process for controllable BS fission is an ongoing problem. In this Letter we investigate the controllable fission of N-soliton beams caused by their reflection at the interface between a Kerr nonlinear medium and a linear dielectric. A variety of effects are shown to occur that may find applications in future all-optical devices based on eigenvalue control.

We study the propagation of light along the z axis of a planar waveguide. The light-field evolution is described by the normalized nonlinear Schrödinger equation:

$$i\frac{\partial q}{\partial \xi} = -\frac{1}{2}\frac{\partial^2 q}{\partial \eta^2} - R(\eta)q(p + |q|^2)\,. \tag{1}$$

We assume that the region $\eta > 0$ of the waveguide is filled with a linear medium with refractive index n_{01}, whereas the region $\eta \le 0$ is occupied by a Kerr nonlinear medium with intensity-dependent refractive index $n = n_{02} + n_2 I$. Let $n_{02} > n_{01}$, so total internal reflection is possible. In that case, $q(\eta, \xi) = (L_{\text{dif}}/L_{\text{nl}})^{1/2} A(\eta, \xi) I_0{}^{-1/2}$. A is the slowly varying envelope, I_0 is the input intensity, $\eta = x/r_0$, r_0 is the beam width, $\xi = z/L_{\text{dif}}$, $L_{\text{dif}} = n_{01}\omega r_0{}^2/c$, $L_{\text{nl}} = 2c/\omega n_2 I_0$, ω is the carrying frequency, $R(\eta) = 1$ for $\eta \le 0$, $R(\eta) = 0$ for $\eta > 0$, and $p = (n_{02}{}^2 - n_{01}{}^2)\omega^2 r_0{}^2/2c^2$. For a laser beam at $\lambda = 1.55\ \mu$m, with $r_0 = 20\ \mu$m and $n_{02} - n_{01} = 3 \times 10^{-4}$, one has $p = 2$. For a nonlinear coefficient, $n_2 \sim 3 \times 10^{-14}\ \text{cm}^2/\text{W}$, $q \sim 1$ corresponds to field intensity $\sim 10\ \text{GW/cm}^2$. We solved Eq. (1) numerically with a split-step Fourier method with meshes dense enough to accurately capture the BS fission. In some cases, to elucidate the soliton properties of the beams reflected at the interface shown in the plots below, we followed their propagation for hundreds of units after the reflection.

We consider a coherent BS of N solitons corresponding to input conditions $q(\eta, 0) = N\,\text{sech}(\eta - \eta_0) \times \exp[i\alpha_{\text{in}}(\eta - \eta_0)]$, where $\eta_0 < 0$ is the initial shift of the beam center. The parameter α_{in} is the small incidence angle measured from the ξ axis ($\alpha_{\text{in}} = 1$ corresponds to ~ 0.01 rad). We assume that the light beam is incident from the nonlinear part of the interface, which is a necessary condition for the existence of a BS. The N-soliton BS is a nonlinear superposition of N single solitons with amplitudes (or form factors) $\chi_k = 2k - 1$ ($k = 1, \ldots, N$) that experience periodic self-compression and expansion as a result of phasing

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and dephasing of the one-soliton components. The strong asymmetric perturbation posed by the interface induces the fission of the BS (Fig. 1). Under suitable conditions, all constituent single solitons survive the interaction and fan out off the interface with reflection angles α_k. Mathematically, such fission of the BS on the interface is related to the breakup of the integrability of Eq. (1) because of the interface. Physically, the fission is due to the spatial filtering caused by the interface. The angle of total internal reflection at the nonlinear interface (measured from the η axis) can be estimated as $\sin\theta = n_{01}/(n_{02} + n_2 I_{\text{ave}})$, where I_{ave} is the averaged beam intensity. The dimensionless spatial frequency Ω_{cr} (or critical value α_{cr} of the incidence angle) corresponding to the total internal reflection is $\Omega_{\text{cr}} = \alpha_{\text{cr}} \approx [2(p + n_{01}{}^2\omega^2 r_0{}^2 n_2 I_{\text{ave}}/c^2)]^{1/2}$. Therefore, in this intuitive picture, the spatial harmonics with frequencies $\Omega \geq \alpha_{\text{cr}}$ can be transmitted into the linear medium, resulting in asymmetric distortion of the BS spectrum. In other words, the reflection at the interface acts as a high-pass filter in the spatial-spectrum domain, with the cutoff frequency depending on beam intensity and guiding parameter p. Hence reflection of the BS at the nonlinear interface has a lot in common with generalized asymmetric spectral filtering of the temporal BS.[6]

Figure 1 illustrates how to control the number of reflected solitons by increasing the incidence angle. For a high enough angle, only the soliton with highest amplitude survives on reflection; its smaller amplitude companions are destroyed by the interaction with the interface, and their energy flow is redistributed between the transmitted beam and the surviving soliton. This redistribution produces narrow output beams. For example, the three-soliton BS with $\alpha_{\text{in}} = 2.7$ transforms after reflection into one beam with amplitude close to 5, which means that the width of the beam is also reduced almost five times.

The total internal reflection and fan-out of single soliton beams at the interface between two nonlinear media is a well-understood process,[12–15] and key results can be connected to the generalized Goos–Hänchen shift acquired by the beams on nonlinear reflection. Figure 2 [row (a)] illustrates the results of simulations of the internal reflection of one-soliton beams, $q(\eta, 0) = \chi\ \text{sech}[\chi(\eta - \eta_0)]\exp[i\alpha_{\text{in}}(\eta - \eta_0)]$, with amplitudes $\chi = 1, 3, 5$. The amplitudes of the reflected solitons decrease monotonically with an increase of the incidence angle. Qualitatively this behavior can be explained with the high-pass-filtering scheme discussed above. The energy flow of the reflected beam can be estimated with a one-soliton spatial spectrum as

$$U = \frac{\pi}{2}\int_{-\infty}^{\alpha_{\text{cr}}} \text{sech}^2\left[\frac{\pi}{2\chi}(\Omega - \alpha_{\text{in}})\right]\text{d}\Omega\,. \tag{2}$$

Assuming that all the reflected energy goes into the soliton beam, one obtains the following estimate for the relative decrease of the form factor:

$$\frac{\chi_{\text{ref}} - \chi}{\chi} \approx \frac{1}{2}\left\{\tanh\left[\frac{\pi}{2\chi}(\alpha_{\text{cr}} - \alpha_{\text{in}})\right] - 1\right\}, \tag{3}$$

which is in reasonable agreement with the results of our numerical simulations. The relative change of the absolute value of the central spatial frequency of reflected soliton α_{ref} for $\alpha_{\text{in}} < \alpha_{\text{cr}}$ can be estimated as

$$\frac{\alpha_{\text{ref}} - \alpha_{\text{in}}}{\alpha_{\text{in}}} \approx -\frac{2}{\pi}\left(\frac{2\chi}{\pi\alpha_{\text{in}}} + \frac{\alpha_{\text{cr}}}{\alpha_{\text{in}}}\right)\exp\left[-\frac{\pi}{2\chi}(\alpha_{\text{cr}} - \alpha_{\text{in}})\right]. \tag{4}$$

Approximation (4) shows that the reflection angle is always smaller than the incidence angle because of the filtering of the high-frequency spatial components [see also Fig. 2, row (a)]. It is also interesting to note that for higher values of the input amplitudes (3 and 5) the

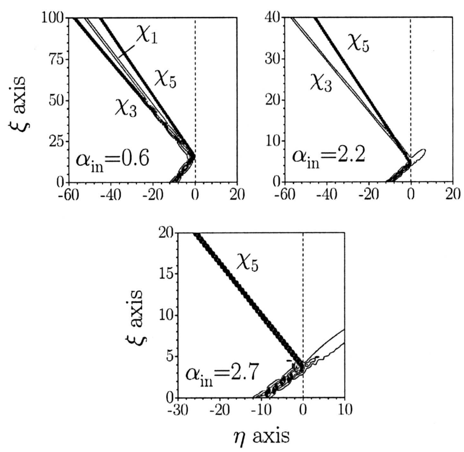

Fig. 1. Dynamics of reflection of three-soliton BSs from the nonlinear interface for various incident angles. χ_k denote solitons from BSs with different amplitudes. $\eta_0 = -10$, $p = 2$.

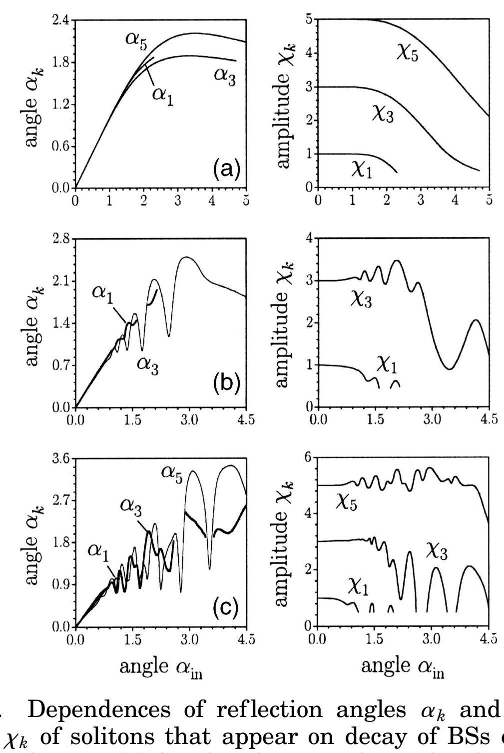

Fig. 2. Dependences of reflection angles α_k and amplitudes χ_k of solitons that appear on decay of BSs on incident angle α_{in}. (a) Single solitons with amplitudes 1, 3, 5; (b) two-soliton BS; (c) three-soliton BS. $\eta_0 = -10$, $p = 2$.

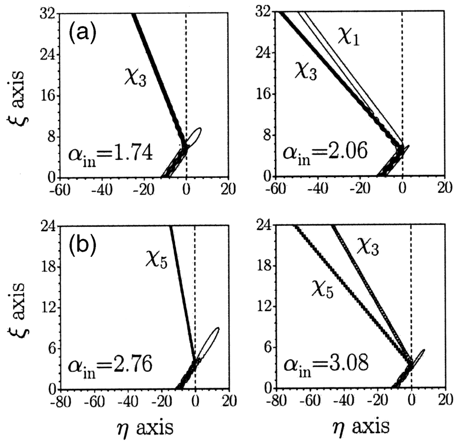

Fig. 3. Dynamics of reflection of (a) two- and (b) three-soliton BSs from the nonlinear interface for close incident angles. χ_k denote solitons from BSs with different amplitudes. $\eta_0 = -10$, $p = 2$.

same reflection angle may correspond to two different angles of incidence.

In contrast, fission of the BS exhibits more complicated behavior [Fig. 2, row (b)]. First, strong modulation of the reflection angle occurs: Small changes of incidence angle α_{in} can generate large variations (up to 30–40%) of reflection angle α_k. This phenomenon can be intuitively understood by taking into account that the precise outcome of the fission is strongly influenced by the exact shape of the BS beam when it hits the interface. For example, if reflection occurs at the point of maximal compression of the BS, the refractive-index difference between the nonlinear medium and the linear dielectric is highest, hence affecting the angles and amplitudes of the reflected beams. If reflection occurs at the point at which the BS is breathing, the refractive-index difference is smallest and more high-frequency components can be transmitted into the linear medium. This phenomenon is much more prominent for a three-soliton BS [Fig. 2, row (c)] because of the larger variation of its intensity profile on propagation toward the reflection point. Depending on the exact propagation length inside the Kerr medium, the peak intensity $|q|^2$ of the three-soliton BS at the reflection point oscillates within the interval 4–64; thus modulation of the reflection angle is correspondingly deep. Moreover, the spatial distribution of the nonlinearly induced refractive index at the reflection region can have a multihumped structure.

Figure 3 illustrates the strong variations of the output angles in the case of two-soliton [row (a)] and three-soliton BSs [row (b)]. Because of the high sensitivity of the reflection angle with the value of the incident angle (10% increase in α_{in} could produce 120% modulation of α_k), this phenomenon might be interesting for switching. We refer to eigenvalue switching, because the soliton propagation angle is related to the real part of the Zakharov–Shabat soliton eigenvalue. In real units, the switching behavior described in Fig. 3(b) corresponds to an increase of separation between reflected soliton channels from 0 up to 0.5 mm after propagation over a few centimeters (the reflection angle for highest-order soliton varies from 0.0072 to 0.0333 rad), on small change in incidence angle (e.g., from 0.0276 to 0.0308 rad). Since, because of their very nature, BSs are also broken by input noise, one can expect better agreement between experimental and theoretical results for relatively high incidence angles, when BSs experience only several pulsations before reflection. It is also worth mentioning that, when $p < 0$, which is not considered here, one could achieve reflection of solitons into nonlinear media only for very small incidence angles, which would require propagation on hundreds of diffraction lengths.

To conclude, we stress that controllable fission of multisoliton complexes of the nonlinear Schrödinger equation at nonlinear interfaces described here offers a new scheme for experimental implementation of the eigenvalue switching concept and has potential applications to future all-optical devices based on soliton signals.

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References

1. A. Hasegawa and Y. Kodama, *Solitons in Optical Communications* (Oxford U. Press, Oxford, 1994).
2. P. K. A. Wai, C. R. Menyuk, Y. C. Lee, and H. H. Chen, Opt. Lett. **11,** 464 (1986).
3. K. Tai, A. Hasegawa, and N. Bekki, Opt. Lett. **13,** 392 (1988).
4. V. V. Afanasyev, V. A. Vysloukh, and V. N. Serkin, Opt. Lett. **15,** 489 (1990).
5. Y. Silberberg, Opt. Lett. **15,** 1005 (1990).
6. S. R. Friberg and K. W. DeLong, Opt. Lett. **17,** 979 (1992).
7. V. A. Vysloukh, V. N. Serkin, and J. R. Taylor, Electron. Lett. **29,** 12 (1993).
8. M. Golles, I. M. Uzunov, and F. Lederer, Phys. Lett. A **231,** 195 (1997).
9. M. Nakazawa, H. Kubota, and K. Tamura, Opt. Lett. **24,** 318 (1999).
10. A. Hasegawa and T. Nyu, J. Lightwave Technol. **11,** 395 (1993).
11. L. Torner, J. P. Torres, and C. R. Menyuk, Opt. Lett. **21,** 462 (1996).
12. A. E. Kaplan, JETP Lett. **24,** 114 (1976).
13. A. E. Kaplan, Sov. Phys. JETP **45,** 896 (1977).
14. P. W. Smith, J.-P. Hermann, W. J. Tomlinson, and P. J. Maloney, Appl. Phys. Lett. **35,** 846 (1979).
15. P. W. Smith and W. J. Tomlinson, IEEE J. Quantum Electron. **20,** 30 (1984).
16. E. Alvarado-Mendez, R. Rojas-Laguna, J. G. Avina-Cervantes, M. Torres-Cisneros, J. A. Andrade-Lucio, J. C. Pedraza-Ortega, E. A. Kuzin, J. J. Sanchez-Mondragon, and V. Vysloukh, Opt. Commun. **193,** 267 (2001).
17. J. Scheuer and M. Orenstein, Opt. Lett. **24,** 1735 (1999).
18. I. Shadrirov and A. A. Zharov, Opt. Commun. **216,** 47 (2003).
19. A. B. Aceves, J. V. Moloney, and A. C. Newell, Phys. Rev. A **39,** 1809 (1989).
20. A. B. Aceves, J. V. Moloney, and A. C. Newell, Phys. Rev. A **39,** 1828 (1989).