Multicolor lattice solitons

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We report on the existence of multicolor solitons supported by periodic lattices made from quadratic nonlinear media. Such lattice solitons bridge the gap between continuous solitons in uniform media and discrete solitons in strongly localized systems and exhibit a wealth of new features. We discovered that, in contrast to uniform media, multipeaked lattice solitons are stable. Thus they open new opportunities for all-optical switching based on soliton packets. © 2004 Optical Society of America

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Since their theoretical prediction in 1988,[1] discrete optical solitons have attracted increasing attention because of their potential for all-optical switching and routing.[2] From the case of cubic media the concept of nonlinear discrete light propagation was extended to quadratic media. Thus families of quadratic discrete solitons in arrays of weakly coupled waveguides have been comprehensively investigated.[3–7] Such strongly localized modes may be used for all-optical switching applications[2,8] with new features compared with the possibilities available with discrete solitons in cubic nonlinear media.[9–12]

A new situation is encountered with lattice solitons initially introduced in Kerr-type nonlinear media.[13–19] Such solitons exist in continuous nonlinear media with imprinted transverse modulation of the refractive index. The concept behind this regime might be termed *tunable discreteness,* with the strength of modulation being the parameter that tunes the system properties from continuous to discrete. Families of lattice solitons were studied in photorefractive crystals with optically induced gratings and in Bose–Einstein condensates. In this Letter we report, for the first time to our knowledge, the existence of lattice solitons in quadratic nonlinear media. Linear stability analysis has revealed the existence of stable families of odd and twisted solitons. Stable twisted solitons can be viewed as a superposition of several solitons, in contrast with multihumped solitons in uniform media that are known to have properties that are unstable.[20]

We start our analysis with the system of coupled nonlinear equations that describe the interaction of fundamental frequency (FF) and second-harmonic (SH) waves under conditions for noncritical type I second-harmonic generation in waveguides:

$$i\frac{\partial q_1}{\partial \xi} = \frac{d_1}{2}\frac{\partial^2 q_1}{\partial \eta^2} - q_1{}^* q_2 \exp(-i\beta\xi) - pR(\eta)q_1\,,$$

$$i\frac{\partial q_2}{\partial \xi} = \frac{d_2}{2}\frac{\partial^2 q_2}{\partial \eta^2} - q_1{}^2 \exp(i\beta\xi) - 2pR(\eta)q_2\,. \qquad (1)$$

Here $q_1 = (2k_1/k_2)^{1/2}[2\pi\omega_0{}^2\chi^{(2)}r_0{}^2/c^2]A_1$ and $q_2 = [2\pi\omega_0{}^2\chi^{(2)}r_0{}^2/c^2]A_2$ are normalized complex amplitudes of the FF and SH waves, $k_1 = k(\omega_0)$, $k_2 = k(2\omega_0) \approx 2k_1$, r_0 is the transverse scale of the input beams, $\eta = x/r_0$, $\xi = z/(k_1 r_0{}^2)$, $\beta = (2k_1 - k_2)k_1 r_0{}^2$ is the phase mismatch, $d_1 = -1$ and $d_2 = -k_1/k_2 \approx -1/2$, and $p = 2\pi\omega_0{}^2 r_0{}^2 \delta\chi^{(1)}/c^2$. The function $R(\eta) = \cos(2\pi\eta/T)$ describes the transverse modulation of the refractive-index profile, where T is the modulation period. In typical quadratic nonlinear crystals, lattice solitons with beam waists of $\sim 15\ \mu$m at wavelengths $\lambda = 1\ \mu$m are formed with peak intensities in the range 0.1–10 GW/cm^2; a refractive-index modulation depth of the order of 10^{-4} corresponds to a guiding parameter $p \sim 1$. The system of Eqs. (1) admits of several conserved quantities, including the energy flow, $U = \int_{-\infty}^{\infty}(|q_1|^2 + |q_2|^2)\mathrm{d}\eta$. Stationary solutions of Eqs. (1) have the form $q_{1,2}(\eta,\xi) = w_{1,2}(\eta)\exp(ib_{1,2}\xi)$, where $w_{1,2}(\eta)$ are real functions and $b_{1,2}$ are real propagation constants that verify $b_2 = \beta + 2b_1$. Lattice soliton families are defined by propagation constant b_1, period of modulation T, and guiding parameter p for a given mismatch β. As one can use scaling transformations $q_{1,2}(\eta,\xi,\beta,p) \to \chi^2 q_{1,2}(\chi\eta,\chi^2\xi,\chi^2\beta,\chi^2 p)$ to obtain various families of lattice solitons from a given family, we selected transverse scale r_0 such that the modulation period verified $T = \pi/2$, and then we varied b_1, β, and p.

There are two types of lowest-order lattice soliton: odd and even. Odd solitons are centered on the lattice sites, and the absolute maximum of their amplitude coincides with the local maximum of $R(\eta)$, whereas even solitons are centered between neighboring lattice sites (see Fig. 1). Even solitons can be viewed as superpositions of in-phase odd solitons. With increasing p, the soliton energy concentrates in neighboring lattice sites; thus lattice solitons approach their discrete counterparts. The energy flow for even solitons exceeds that for odd solitons [Fig. 1(a)]. There are equal lower cutoffs for the existence of odd and even solitons.

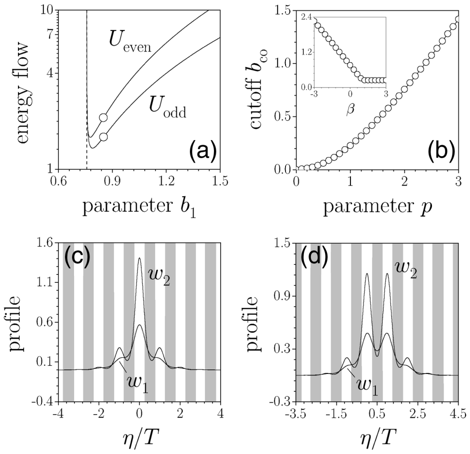

Fig. 1. (a) Energy flow versus propagation constant for even and odd lattice solitons. The vertical dashed line in (a) stands for cutoff. (b) Cutoff of the propagation constant for odd and even solitons as a function of guiding parameter at $\beta = 0$. Inset, cutoff versus phase mismatch at $p = 2$. Profiles of odd (c) and even (d) solitons correspond to points marked by circles in (a). (a), (c), (d) $p = 2$ and $\beta = 0$. (c), (d) In white regions $R(\eta) \geq 0$; in gray regions $R(\eta) < 0$.

Cutoff value b_{co} is a monotonically increasing function of p [Fig. 1(b)], and $b_{co}(\beta) = \max[b_0 - (\beta - \beta_0)/2; b_0]$, where b_0 is the cutoff at $\beta \rightarrow \infty$ and β_0 is a mismatch shift that is due to the lattice. Thus the lattice affects the energy sharing between FF and SH waves. For example, the fraction of total energy flow carried by the SH wave increases with lattice depth. Both b_0 and β_0 increase with growing p. The functions $b_0(p)$ and $\beta_0(p)$ are analogous to $b_{co}(p)$ shown in Fig. 1(b). At $\beta < \beta_0$ the soliton energy flow diverges as $b_1 \rightarrow b_{co}$, whereas at $\beta \geq \beta_0$ it vanishes as $b_1 \rightarrow b_{co}$. At the cutoff point either the SH wave or both the FF and the SH waves are transformed into linear Bloch waves.

We found various families of higher-order lattice solitons that can be considered as combinations of several in-phase and out-of-phase odd solitons. Here we concentrate only on solitons for which $w_1(\eta)$ changes its sign for neighboring lattice sites (out-of-phase combinations) because we found in-phase combinations to be unstable. Higher-order modes sustained by combinations of out-of-phase odd lattice solitons have much in common with twisted discrete modes. The simplest twisted lattice solitons, which correspond to combinations of two (first twisted) and three (second twisted) odd solitons, are described in Fig. 2. Their energy flow is a nonmonotonic function of the propagation constant [Fig. 2(a)]. There is a narrow region near cutoff, not even visible in the figure, where $dU/db_1 < 0$. The cutoff for the first twisted soliton versus β and p is shown in Fig. 2(b). Notice that twisted solitons are not transformed into Bloch waves at cutoff. Typical profiles of twisted solitons are depicted in Figs. 2(c) and 2(d).

To elucidate the stability of the lattice soliton families, we searched for perturbed solutions of Eqs. (1) in the form $q_{1,2}(\eta, \xi) = [w_{1,2}(\eta) + U_{1,2}(\eta, \xi) + iV_{1,2}(\eta, \xi)]\exp(ib_{1,2}\xi)$. We looked for exponentially growing perturbations $U_{1,2}(\eta, \xi) = \mathrm{Re}[u_{1,2}(\eta, \delta) \times \exp(\delta\xi)]$ and $V_{1,2}(\eta, \xi) = \mathrm{Re}[v_{1,2}(\eta, \delta)\exp(\delta\xi)]$ with complex growth rate δ. We found odd solitons to be unstable in a narrow band of propagation constants near cutoff b_{co} for $\beta < \beta_0$ [Fig. 3(a)]. The corresponding growth rates are purely real and are associated with exponential instabilities. For $\beta \geq \beta_0$, odd solitons are stable in the entire domain of their

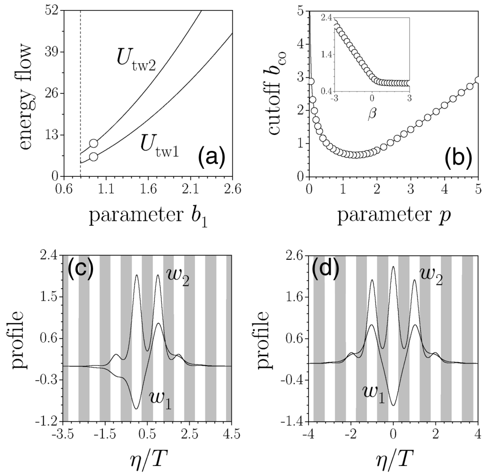

Fig. 2. Energy flow versus propagation constant for first and second twisted lattice solitons. The vertical dashed line in (a) stands for cutoff. (b) Cutoff of the propagation constant for the first twisted soliton as a function of guiding parameter at $\beta = 0$. Inset, cutoff versus phase mismatch at $p = 2$. Profiles of first (c) and second (d) twisted solitons correspond to points marked by circles in (a). (a), (c), (d) $p = 2$ and $\beta = 0$.

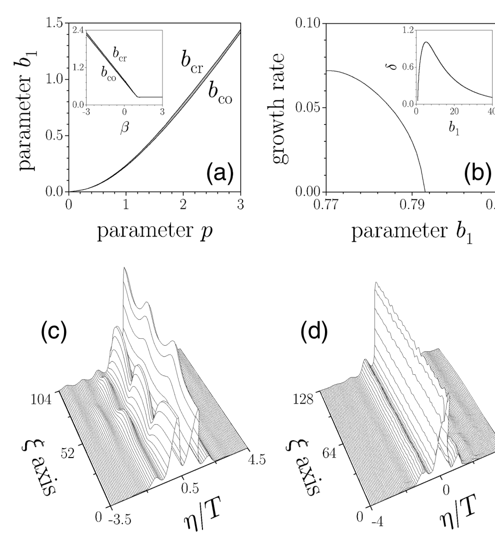

Fig. 3. (a) Areas of stability and instability (shaded) for an odd soliton on the (b_1, p) plane at $\beta = 0$. Inset, the same areas on the (b_1, β) plane at $p = 2$. (b) Perturbation growth rate for an odd soliton versus propagation constant at $\beta = 0$ and $p = 2$. Inset, growth rate for an even soliton for the same parameters. (c) Perturbation-induced decay of an even soliton with $b_1 = 0.85$ at $\beta = 0$ and $p = 2$. (d) Stable propagation of an odd soliton with $b_1 = 0.85$ at $\beta = 0$ and $p = 2$ perturbed with white noise with variance ${\sigma_{1,2}}^2 = 0.02$. (c), (d) Only the SH wave is shown.

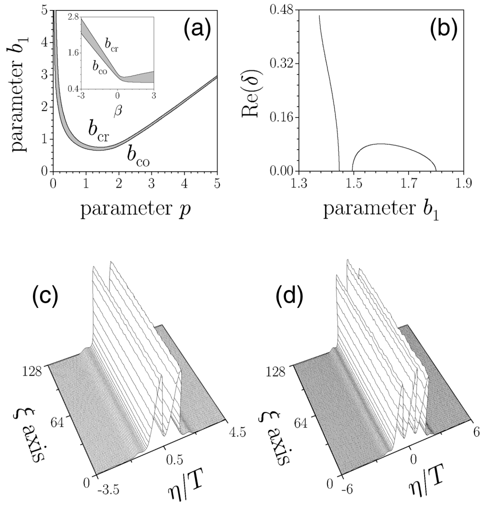

Fig. 4. Areas of stability and instability (shaded) for the first twisted soliton on the (b_1, p) plane at $\beta = 0$. Inset, same areas on the (b_1, β) plane at $p = 2$. (b) Real part of the perturbation growth rate for the first twisted soliton versus propagation constant at $\beta = 0$ and $p = 0.3$. Stable propagation of (c) the first and (d) the second twisted solitons with $b_1 = 1.2$ at $\beta = 0$ and $p = 2$ in the presence of white input noise with variance $\sigma_{1,2}{}^2 = 0.02$. (c), (d) Only the SH wave is shown.

existence. At $\beta < \beta_0$ the growth rate for odd solitons was found to decrease with increasing propagation constant; at $b_1 = b_{\rm cr}$ odd solitons became stable [Fig. 3(b)]. Figure 3(d) shows the stable propagation of an odd soliton in the presence of input white noise added to the soliton profile. Even lattice solitons were found to be exponentially unstable in the entire domain of their existence. When $U \to \infty$, an even soliton is transformed into two narrow solitons located in neighboring lattice sites and $\delta \to 0$ [Fig. 3(b), inset]. Typical decay dynamics of an even soliton are shown in Fig. 3(c). A small perturbation of the input profile causes oscillations of the field amplitude in the neighboring lattice sites.

A central finding of this Letter is that twisted lattice solitons become completely stable when their energy flow exceeds a threshold (or when $b_1 \geq b_{\rm cr}$), i.e., in almost the whole domain of their existence [Fig. 4(a)]. The real part of the perturbation growth rate for the first twisted soliton versus propagation constant is shown in Fig. 4(b). Notice that the left branch corresponds to purely exponential instabilities and the right branch corresponds to oscillatory instabilities [Re(δ)Im(δ) $\neq 0$]. Separate windows where Re(δ) $= 0$ exist, even inside the shaded areas in Fig. 4(a) (which show maximum value of propagation constant for stabilization), but we do not study them here. Results of linear stability analysis were confirmed by direct numerical integration of Eqs. (1). For example, Figs. 4(c) and 4(d) illustrate stable propagation of the first and second twisted solitons when they are perturbed by significant initial random noise. We also verified that twisted lattice solitons can be excited with FF-only beams that contain several oscillations of the field in the transverse direction, with a periodicity two times higher than that of the lattice. Such input beams can be created, e.g., by the interference of plane waves.

To conclude, we stress the existence of stable combinations of out-of-phase twisted multicolor solitons in quadratic lattices, in contrast to the multipeaked solitons in uniform media that are all known to be unstable. The discovery of the stability of such multipeaked lattice solitons immediately suggests the important possibility of constructing and manipulating stable soliton packets beyond single soliton bits, a property that might open a new door in all-optical switching schemes.

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