Parametric amplification of soliton steering in optical lattices

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We report on the effect of parametric amplification of spatial soliton swinging in Kerr-type nonlinear media with longitudinal and transverse periodic modulation of the linear refractive index. The parameter areas are found where the soliton center motion is analogous to the motion of a parametrically driven pendulum. This effect has potential applications for controllable soliton steering.

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The effect of spatially periodic linear refractive-index variations on the propagation of solitons is of great importance for applied photonics.[1] In particular, nonlinear Kerr-type media with shallow periodic refractive-index gratings support lattice solitons that bridge the gap between the continuous and the discrete solitons. Competition between the characteristic scales of the problem, namely, the beam width and the lattice period, leads to a variety of propagation situations. For example, radiative destruction of a soliton moving along the lattice and periodic particlelike oscillations in a broad external potential were predicted[2] as well as the possibility of radiative soliton trapping and switching.[3]

In arrays of weakly coupled waveguides, spatial solitons were predicted in Ref. 4 and experimentally observed in Refs. 5 and 6. Such discrete solitons proved to be useful for a number of practical applications, including all-optical switching and power-dependent soliton steering.[1,7–12] Although by its nature the phenomenon of soliton trapping in a periodic lattice is analogous to that in discrete waveguide arrays, periodic lattices offer additional possibilities for control of soliton-based device operation. Recently spatial solitons were analyzed theoretically and demonstrated experimentally in optically induced lattices in both one and two transverse dimensions.[13–16] In this Letter we report the amplification of a spatial soliton swinging in a guiding channel of the lattice caused by a small longitudinal modulation of the linear refractive index.

We start our analysis with the nonlinear Schrödinger equation describing light propagation in a slab waveguide with a focusing Kerr-type nonlinearity. The laser beam is allowed to diffract along the x axis only, and the linear refractive index is modulated along both the transverse x and the longitudinal z axes:

$$i\frac{\partial q}{\partial \xi} = -\frac{1}{2}\frac{\partial^2 q}{\partial \eta^2} - q|q|^2 - pQ(\xi)R(\eta)q\,. \qquad (1)$$

Here $q(\eta, \xi) = (L_{\mathrm{dif}}/L_{\mathrm{nl}})^{1/2}A(\eta, \xi)I_0^{-1/2}$, $A(\eta, \xi)$ is the slowly varying envelope, I_0 is the input peak intensity, $\eta = x/r_0$, r_0 is the input beam width, $\xi = z/L_{\mathrm{dif}}$, $L_{\mathrm{dif}} = n_0\omega r_0^2/c$ is the diffraction length, $L_{\mathrm{nl}} = 2c/\omega n_2 I_0$ is the nonlinear length, ω is the carrying frequency, $p = L_{\mathrm{dif}}/L_{\mathrm{ref}}$ is the guiding parameter, $L_{\mathrm{ref}} = c/(\delta n\omega)$ is the linear refraction length, and δn is the modulation depth of the index of refraction. The variation of the refractive index along the longitudinal axis is described by the function $Q(\xi) = 1 - \mu\cos(\Omega_\xi \xi)$, where $|\mu| < 1$. We consider two types of refractive-index profile: harmonic, $R(\eta) = \cos(\Omega_\eta \eta)$, and parabolic, $R(\eta) = 1 - (\Omega_\eta \eta)^2/2$, with Ω_η being the modulation frequency. We assume that the depth of the transverse modulation is small compared with the unperturbed index n_0 and is of the order of the nonlinear correction due to the Kerr effect. The longitudinal modulation is weak and smooth, so that reflected waves are negligible.

First, we analyze the soliton propagation with the aid of the effective-particle approach,[2,3] based on the equation for the integral beam center, $\langle\eta\rangle = (1/U)\int_{-\infty}^{\infty}|q|^2\eta\,\mathrm{d}\eta$. One readily finds that the equation of motion for $\langle\eta\rangle$ is given by

$$\frac{\mathrm{d}^2}{\mathrm{d}\xi^2}\langle\eta\rangle = p\,\frac{Q(\xi)}{U}\int_{-\infty}^{\infty}|q|^2\frac{\mathrm{d}R}{\mathrm{d}\eta}\,\mathrm{d}\eta\,, \qquad (2)$$

where $U = \int_{-\infty}^{\infty}|q|^2\mathrm{d}\eta$ is the energy flow, or the effective mass, that remains constant upon propagation. In this approach, on the right-hand side of Eq. (2) one assumes that the beam shape is unchanged upon propagation in a doubly periodic nonlinear lattice. We set $q(\eta, \xi) = q_0\,\mathrm{sech}[\chi(\eta - \langle\eta\rangle)]\exp[i\alpha(\eta - \langle\eta\rangle) + i\phi]$, where q_0 is the amplitude, χ is the form factor or inverse beam width, α is the slope of the phase front (or effective-particle velocity), and ϕ is the phase. One has $q_0 \approx \chi$ in two limiting cases of narrow ($\Omega_\eta \ll \chi$) and wide ($\Omega_\eta >> \chi$) soliton beams.[3] Substitution of this expression into Eq. (2) yields the equation of motion of a parametrically driven pendulum:

$$\frac{\mathrm{d}^2}{\mathrm{d}\xi^2}\langle\eta\rangle + [1 - \mu\cos(\Omega_\xi\xi)]\frac{\Omega_0^2}{\Omega_\eta}\sin(\Omega_\eta\langle\eta\rangle) = 0\,, \qquad (3)$$

where $\Omega_0 = [p\Omega_\eta^2(\pi\Omega_\eta/2\chi)\sinh^{-1}(\pi\Omega_\eta/2\chi)]^{1/2}$ defines the frequency of small-amplitude oscillations of the soliton center at $\mu = 0$. We assume that the beam

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is launched in the point $\eta = 0$, so that $\langle\eta\rangle_{\xi=0} = 0$ and $(\mathrm{d}\langle\eta\rangle/\mathrm{d}\xi)_{\xi=0} = \alpha_0$, where α_0 is the incident angle. For small amplitude oscillations in a doubly periodic lattice $\Omega_\eta\langle\eta\rangle \ll 1$, and in the media with a parabolic refractive-index profile Eq. (3) reduces to the Mathieu equation.

For $\mu = 0$ the soliton beam can be considered as an effective particle moving in a harmonic potential. With an increase in the incident angle the kinetic energy of the particle increases. For low kinetic energy the particle remains located in the central potential well. The particle escapes from the central well and starts to walk along the structure when the kinetic energy exceeds the height of the potential barrier. The critical value of the incident angle dividing regions of finite and infinite motion is given by $\alpha_{\mathrm{cr}} = 2\Omega_0/\Omega_\eta$, whereas trajectories of the beam center are described by $\langle\eta\rangle = (2/\Omega_\eta)\arcsin[m\ \mathrm{sn}(\Omega_\eta\alpha_{\mathrm{cr}}\xi/2, m)]$ for $\alpha_0 < \alpha_{\mathrm{cr}}$ ($m = \alpha_0/\alpha_{\mathrm{cr}}$) and by $\langle\eta\rangle = (2/\Omega_\eta)\arcsin[\mathrm{sn}(\Omega_\eta\alpha_0\xi/2, m)]$ for $\alpha_0 > \alpha_{\mathrm{cr}}$ ($m = \alpha_{\mathrm{cr}}/\alpha_0$). In the limit $\alpha_0 \ll \alpha_{\mathrm{cr}}$ the first of these equations describes harmonic oscillations $\langle\eta\rangle = (\alpha_0/\Omega_0)\sin(\Omega_0\xi)$, whereas in the limit $\alpha_0 >> \alpha_{\mathrm{cr}}$ the second equation describes the motion with the constant velocity $\langle\eta\rangle = \alpha_0\xi$. As $\alpha_0 \to \alpha_{\mathrm{cr}}$ oscillations become nonlinear, and according to Eq. (3) the frequency of free oscillations transforms as $\Omega_0 \to \Omega_0/[4K(m)]$, where $K(m = \alpha_0/\alpha_{\mathrm{cr}})$ is the elliptical integral of the first kind. It means that Ω_0 rapidly decreases as $\alpha \to \alpha_{\mathrm{cr}}$.

In the presence of a longitudinal refractive-index modulation, i.e., $\mu \neq 0$, Eq. (3) predicts swinging of the effective particle with exponentially growing amplitude, provided that the resonance condition $\Omega_\xi = 2\Omega_0$ is satisfied. With an increase in the oscillation amplitude the instantaneous oscillation frequency drops off and the system removes itself from the condition of parametric resonance. This should lead to a decrease in the amplitude of oscillations that is accompanied by a rise in the instantaneous frequency. Therefore resonance conditions can be met periodically in ξ. In the case of the parabolic transverse refractive-index profile the frequency of particle oscillations $\Omega_0 = p^{1/2}\Omega_\eta$ does not depend on the oscillation amplitude; parametric resonance condition $\Omega_\xi = 2\Omega_0$ is never violated, and oscillation amplitude increases steadily.

The actual soliton propagation dynamics in the longitudinally periodic lattices can depart from that predicted by the effective-particle approach. One cause is that the analytic approach does not take into account radiative losses that occur when a high-frequency wing of the soliton spatial spectrum is transmitted through the potential barrier and leaves the guiding channel. The higher the incidence angle, the more significant the fraction of soliton energy that is lost. Moreover, strong longitudinal and transverse variations of the refractive index can even destroy the soliton or considerably change its energy. We have found, however, that there exists a wide range of parameters for which the soliton behavior almost exactly matches the effective-particle model. To do so, we solved Eq. (1) numerically with the initial condition $q|_{\xi=0} = \chi\ \mathrm{sech}(\chi\eta)\exp(i\alpha_0\eta)$, where $\alpha_0 \sim 0.01\alpha_{\mathrm{cr}}$. Upon definition of α_{cr} we used the same expression, $\alpha_{\mathrm{cr}} = 2\Omega_0/\Omega_\eta$, for both a doubly periodic lattice and a medium with a parabolic refractive-index profile, where frequency $\Omega_\xi \approx 2\Omega_0$ was chosen according to the type of the medium. The typical particlelike soliton propagation in the doubly periodic lattice is shown in Fig. 1(a). Figure 1(b) depicts the soliton propagation in the medium with the parabolic refractive-index profile $R(\eta) = 1 - (\Omega_\eta\eta)^2/2$ that is modulated harmonically along the ξ axis. Notice the steady increase in oscillation amplitude in the medium with a parabolic refractive-index profile. Upon numerical integration we followed the evolution of the integral beam center $\langle\eta\rangle$ and instantaneous angle $\alpha = \mathrm{d}\langle\eta\rangle/\mathrm{d}\xi$. In a doubly periodic lattice the maximal value of instantaneous angle $\alpha_{\max}$ is reached at a finite distance $\xi_{\max} < \xi_{\mathrm{b}}$ [see Fig. 2(a)]. The dependence of $\alpha_{\max}$ on normalized frequency detuning $\nu = (\Omega_\xi - 2\Omega_0)/2\Omega_0$ is shown in Fig. 2(b). This dependence has the form of a classical asymmetric resonance curve for an oscillator with a soft nonlinearity. Notice the jump in the maximal angle that occurs with an increase in detuning—a feature typical for resonance curves of nonlinear parametrically driven systems.

Figure 2(c) illustrates the dependence of angle $\alpha_{\max}$ on guiding parameter p. Numerical simulations revealed that at resonance frequency $\nu = 0$ and for $p \geq 0.1$ the system meets conditions of saturation of the parametric gain when instantaneous angle α reaches the level of $\sim 0.6\alpha_{\mathrm{cr}}$. Therefore at $p \geq 0.1$ one has $\alpha_{\max} \sim p^{1/2}$. Figure 2(d) shows propagation distance $\xi_{\max}$ where the maximal value of the

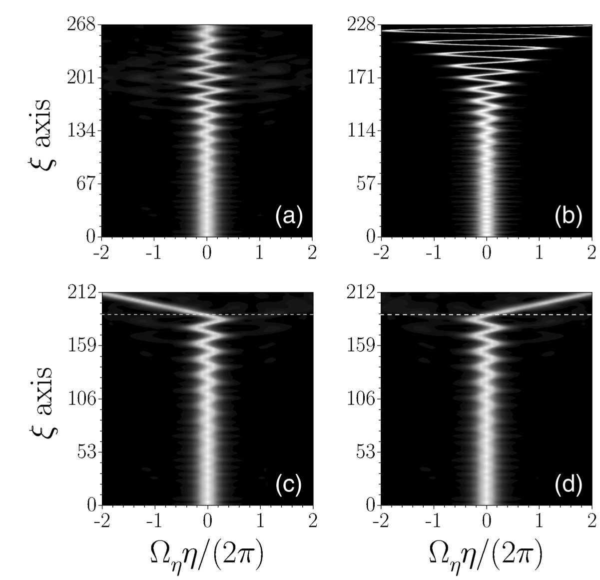

Fig. 1. Parametric amplification of spatial soliton swinging (a) in a doubly periodic lattice and (b) in a waveguide with a parabolic refractive-index profile. Incident angle $\alpha_0 = 0.01\alpha_{\mathrm{cr}}$. In (c) and (d) the possibility of manipulation of the output angle at the entrance of a doubly periodic lattice is shown. In (c) $\alpha_0 = 0.01\alpha_{\mathrm{cr}}$, whereas in (d) $\alpha_0 = -0.01\alpha_{\mathrm{cr}}$. Dashed lines in (c) and (d) show the boundary between the uniform medium and the medium with periodic modulation of the refractive index. For all plots, $p = 0.25$, $\mu = 0.25$, $\boldsymbol{\Omega_\eta} = 1$, $\chi = 1$, and condition $\Omega_\xi = 2\Omega_0$ of the parametric resonance is satisfied.

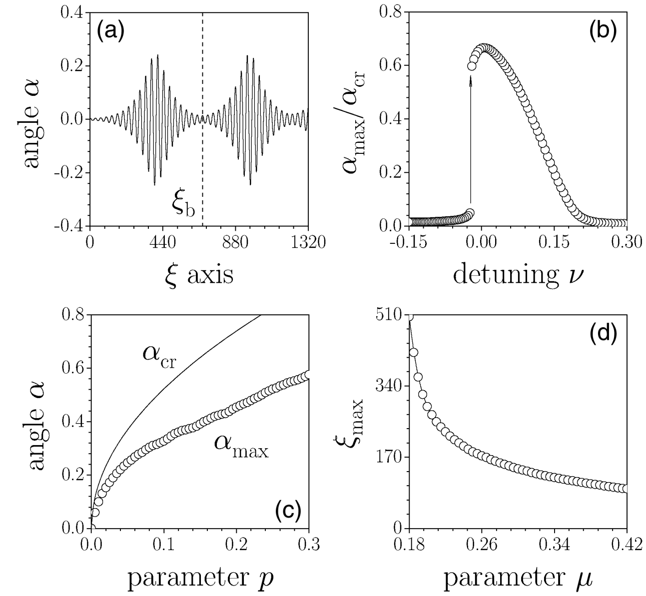

Fig. 2. (a) Dependence of the instantaneous angle on propagation distance at $p = 0.05$ and $\mu = 0.25$. (b) Maximal propagation angle versus detuning at $p = 0.25$ and $\mu = 0.25$. (c) Critical and maximal propagation angles versus guiding parameter at $\mu = 0.25$. (d) Dependence of the distance ξ_{max} on the longitudinal refractive-index modulation depth at $p = 0.25$. For all plots, $\Omega_\eta = 1$ and $\chi = 1$. In (a), (c), and (d) the parametric resonance condition $\Omega_\xi = 2\Omega_0$ is satisfied.

instantaneous angle is reached versus refractive-index modulation depth μ along the ξ axis. In the interval $0.18 \le \mu \le 0.42$, distance ξ_{max} decreases exponentially with increasing modulation depths, whereas α_{max} remains unchanged. For higher values of μ the soliton motion becomes complicated and irregular. The considerable increase in the amplitude of the soliton oscillations is possible only in the case of $\Omega_\eta/\chi \le 1$. For $\Omega_\eta/\chi >> 1$ propagation of the bell-shaped input beam is accompanied by strong reshaping, resulting in formation of a lattice soliton that covers several lattice sites, whereas α_{max} quickly drops off with increasing Ω_η/χ. The same phenomenon occurs for higher values of guiding parameter $p \gtrsim 0.5$ and longitudinal modulation depth $\mu \gtrsim 0.5$, thus defining the approximate range of applicability of the effective-particle model and sech-type representation for a soliton profile.

The phenomenon of parametric swinging uncovered here can be used for controllable soliton steering. Such a possibility is highlighted in the top parts of Figs. 1(c) and 1(d), which display the light evolution for the configuration where the segment of the nonlinear medium with periodic modulation of the refractive index is followed by a uniform nonlinear medium. The plots show that the small variations in the incident angle are parametrically amplified and result in considerable changes in the output angle: Plots depict the soliton propagation launched into the nonlinear grating at small but opposite angles $\alpha_0 = \pm 0.01\alpha_{cr}$. A similar wide-range tuning of the output angle is also achieved by varying the depth of the modulation of the linear refractive index along the ξ axis.

In conclusion, we have discovered that, under proper conditions, longitudinally periodic lattices can exhibit amplification of soliton swinging, a phenomenon that enriches the possibilities of soliton control in the lattices and might find direct applications to controllable soliton steering. The predicted effect should be observable in photorefractive crystals in which longitudinal modulation can be created by a periodic background illumination whose intensity can be adjusted to control the depth of the modulation. A transverse periodic grating can be produced by two interfering plane waves.[15,16] Numerical simulations indicate that even considerable (up to 20% in intensity) white input noise does not affect the process of parametric soliton swinging.

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