Packing, unpacking, and steering of multicolor solitons in optical lattices

Yaroslav V. Kartashov,* Victor A. Vysloukh,† and Lluis Torner

ICFO–Institut de Ciencies Fotoniques and Department of Signal Theory and Communications, Universitat Politecnica de Catalunya, 08034 Barcelona, Spain

Received January 14, 2004

We discuss potential applications of multicolor solitons supported by periodic lattices imprinted in quadratic nonlinear media. Such lattice solitons can be packed together with appropriate relative phases to form stable soliton trains that can be treated as bit sequences. We describe controllable splitting of the trains into their soliton constituents and the angle- and power-controlled steering and trapping of solitons moving across the lattice into the desired guiding channel. © 2004 Optical Society of America

OCIS codes: 190.5530, 190.4360, 060.1810.

The phenomenon of guidance and the division of light in arrays of coupled optical waveguides have gained growing attention since the theoretical prediction of discrete solitons.[1] Discrete solitons exist in weakly coupled waveguide arrays and are formed by competition of nonlinearity and discrete diffraction.[2] The concept of discrete light propagation was established for both cubic and quadratic nonlinear media, and the general properties of discrete solitons are now well established.[1–11]

Harmonic lattices with cubic[12–17] and quadratic[18] nonlinearity also support stable solitons, and their properties can depart considerably from the properties of the usual discrete solitons. Despite the fact that neighboring waveguides in a harmonic lattice are always strongly coupled, by changing the depth of linear refractive-index modulation one can tune the system's properties from continuous to discrete. Besides lowest-order solitons, optical lattices support tightly packed stable multihumped structures[18] or soliton trains that are highly unstable in a uniform medium in the absence of a lattice[19] and analogous to twisted local modes in discrete systems.

In the present Letter we explore a new concept for application of lattice solitons: packing of an arbitrary number of single solitons with appropriately engineered phases into stable trains in quadratic nonlinear media and controllable unpacking of trains into its constituents that have various escape angles. We address the possibility of excitation of such trains by truncated interference patterns and soliton steering.

We start our analysis with the system of coupled nonlinear equations that describes the interaction of fundamental frequency (FF) and second-harmonic (SH) waves under conditions for noncritical type I SH generation in waveguides:

$$i\,\frac{\partial q_1}{\partial \xi} = \frac{d_1}{2}\,\frac{\partial^2 q_1}{\partial \eta^2} - {q_1}^* q_2 \exp(-i\beta\xi) - pR(\eta)q_1\,,$$

$$i\,\frac{\partial q_2}{\partial \xi} = \frac{d_2}{2}\,\frac{\partial^2 q_2}{\partial \eta^2} - {q_1}^2 \exp(i\beta\xi) - 2pR(\eta)q_2\,. \qquad (1)$$

Here $q_1 = (2k_1/k_2)^{1/2}[2\pi\omega_0{}^2\chi^{(2)}r_0{}^2/c^2]A_1$ and $q_2 = [2\pi\omega_0{}^2\chi^{(2)}r_0{}^2/c^2]A_2$ are normalized complex amplitudes of the FF and SH waves; $k_1 = k(\omega_0)$; $k_2 = k(2\omega_0) \approx 2k_1$; r_0 is the transverse scale of the input beams; $\eta = x/r_0$; $\beta = (2k_1 - k_2)k_1r_0{}^2$ is the phase mismatch; $d_1 = -1$; $d_2 = -k_1/k_2 \approx -1/2$; $p = 2\pi\omega_0{}^2r_0{}^2\delta\chi^{(1)}/c^2$; and the function $R(\eta) = \cos(2\pi\eta/T)$ describes transverse modulation of the refractive index, where T is the modulation period. Energy flow $U = \int_{-\infty}^{\infty}(|q_1|^2 + |q_2|^2)\mathrm{d}\eta$ is one of the conserved quantities of Eqs. (1).

We search for solutions of Eqs. (1) of the form $q_{1,2}(\xi,\eta) = w_{1,2}(\eta)\exp(ib_{1,2}\xi)$, where $w_{1,2}(\eta)$ are real functions and $b_{1,2}$ are real propagation constants that verify $b_2 = \beta + 2b_1$. Further, we selected transverse scale r_0 such that modulation period $T = \pi/2$, and then we varied b_1, β, and p. For linear stability analysis we considered perturbed solutions $q_{1,2}(\eta,\xi) = [w_{1,2}(\eta) + U_{1,2}(\eta,\xi) + iV_{1,2}(\eta,\xi)]\exp(ib_{1,2}\xi)$, where real and imaginary parts of perturbation could grow with the complex growth rate. Linearization yields the eigenvalue problem that was solved numerically. A detailed description of lattice soliton properties was reported recently[18]; thus here we concentrate only on odd and the simplest twisted solitons (Fig. 1). The absolute amplitude maximum of the odd soliton coincides with the local maximum of $R(\eta)$ [Fig. 1(b)], whereas twisted solitons can be viewed as a superposition of several odd solitons with opposite phases; i.e., function $w_1(\eta)$ changes its sign for neighboring lattice sites [Fig. 1(d)].

Stability analysis shows that the narrow instability band exists near lower cutoff for odd solitons [Fig. 1(a)]; it disappears when phase mismatch β exceeds a certain critical level. Important results of this Letter are that stabilization above a certain threshold energy level takes place also for twisted solitons of arbitrary higher orders [Fig. 1(c)] and that the threshold value of the propagation constant for stabilization increases only slightly with the growth of the soliton order. This result implies the possibility of packing tightly a desired number of individual lattice solitons into a robust soliton train that survives in the presence of random perturbations.[18]

From an applied point of view the important issue is the formation of stable twisted solitons or soliton trains. We have found that such solitons can be

0146-9592/04/121399-03$15.00/0

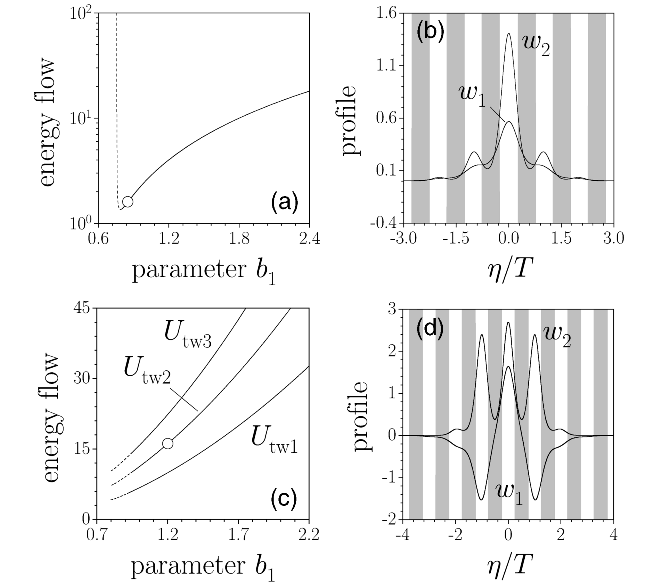

Fig. 1. (a) Energy flow versus propagation constant for an odd lattice soliton. (b) Profile of an odd soliton corresponding to the point marked by a circle in (a). (c) Energy flow versus propagation constant for twisted solitons of lowest orders. (d) Profile of the second twisted soliton, corresponding to the point marked by a circle in (c). (a), (c) Dashed curves, unstable branches; solid curves, stable branches. (b), (d) Shaded regions, $R(\eta) \leq 0$; unshaded regions, $R(\eta) > 0$. $p = 2$, $\beta = 0$, $T = \pi/2$.

excited with FF beams that contain several oscillations of the field in the transverse direction with a periodicity that is two times higher than that of the lattice. Figures 2(a) and 2(b) show excitation of a second twisted soliton for input conditions $q_1|_{\xi=0} = A_1 \cos(\pi\eta/T)$ at $|\eta| \leq 3T/2$ and $q_1|_{\xi=0} = 0$ at $|\eta| > 3T/2$, while $q_2|_{\xi=0} \equiv 0$. The corresponding input field distribution can be formed by the interference of plane waves (truncated interference pattern). In such a case the excitation efficiency of lattice soliton trains is sufficiently high: $U_{\mathrm{out}}/U_{\mathrm{in}} \sim 0.6$, where U_{out} is the energy flow of a soliton and U_{in} is the input energy flow.

Notice that odd solitons survive encountering such perturbations as lattice tilts [Fig. 2(c)]. At the border between straight and tilted lattices an odd soliton loses part of its energy and changes its propagation direction to that dictated by the tilted lattice. Naturally that energy loss increases with growth of the tilt angle [Fig. 2(d)] and, when that angle exceeds the critical angle, the soliton is destroyed. We found that soliton trains also survive propagation through lattice tilts if $\alpha_{\mathrm{tilt}} \lesssim 0.5$.

In tightly packed trains individual solitons are held together only by the periodic lattice. Removal of the lattice causes fast splitting (or unpacking) of the train, because in twisted solitons the interaction between neighboring constituents is repulsive. One can use such splitting to obtain a set of diverging solitons with controllable escape angles [Figs. 3(a) and 3(b)]. Further, we calculated escape angles for the outermost solitons that appear on decay of the first and the second twisted solitons. The escape angle increases with the growth of the twisted soliton order and is the nonmonotonic function of an overall train energy flow [Fig. 3(c)]. The decrease of α_{esc} at $U \to \infty$ is related to diminishing overlap of tails. The escape angle grows monotonically with an increase in the guiding parameter [Fig. 3(d)]. Because the interaction of neighboring solitons in the train is

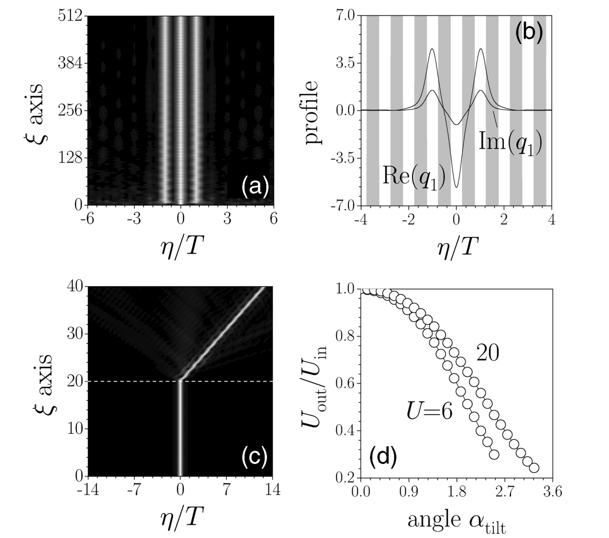

Fig. 2. (a) Excitation of the second twisted lattice soliton and (b) real and imaginary parts of the FF wave field at $\xi = 512$. At $\xi = 0$, FF wave amplitude $A_1 = 8$. (c) Change of propagation direction of the odd soliton with $U = 6$ on entrance to a tilted lattice with $\alpha_{\mathrm{tilt}} = 1$. Dashed line, border between straight and tilted lattices. (d) Efficiency of energy transmission for odd solitons versus lattice tilt angle. In (a) and (c), only the SH wave is shown. $\beta = 0$, $p = 2$, $T = \pi/2$.

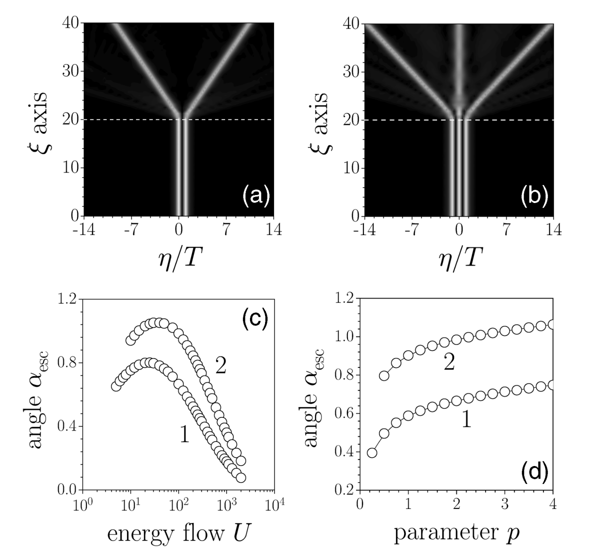

Fig. 3. Decay of (a) the first and (b) the second twisted solitons at the boundary between a uniform medium and a medium with periodic modulation of the refractive index. Horizontal dashed lines, boundaries between two media. Energy flow, $U = 50$; guiding parameter, $p = 2$. Only the FF wave is shown. Escape angles for solitons that appear on decay of 1, the first and 2, the second twisted solitons (c) versus energy flow at $p = 2$ and (d) versus the guiding parameter at $U = 100$. $\beta = 0$, $T = \pi/2$.

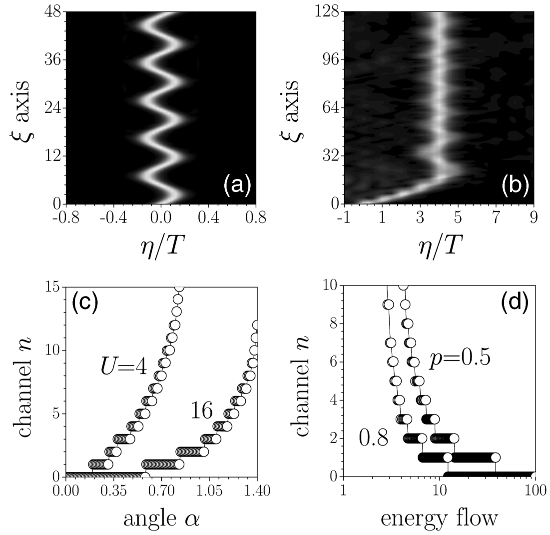

Fig. 4. (a) Periodic oscillations of an odd soliton with $U = 10$ within a central waveguide at $p = 2$, $\alpha = 1$, $T = 4\pi$. (b) Fast switching of an odd soliton with $U = 10$ into a fourth lattice channel at $p = 0.25$, $\alpha = 0.62$, $T = \pi/2$. In (a) and (b), only the SH wave is shown. (c) Dependence of the number of output channels on incident angle for different energy flows at $p = 0.5$, $T = \pi/2$. (d) Dependence of the output channel number on energy flow for different guiding parameters at $\alpha = 0.8$, $T = \pi/2$. Phase mismatch, $\beta = 0$.

repulsive, the splitting process is almost unaffected by noise, as was confirmed by extensive simulations in the presence of broadband input noise with variance up to ${\sigma_{1,2}}^2 = 0.05$. The splitting shown in Figs. 3(a) and 3(b) is reversible; i.e., spatial solitons launched into homogeneous quadratic media with appropriately adjusted relative phases and incidence angles can be tightly packed into the lattice and can propagate further as a compact soliton train.

Multicolor lattice solitons permit energy- and angle-controlled steering. To illustrate that this is so, we solved Eqs. (1) for input conditions $q_1|_{\xi=0} = w_1(\eta)\exp(i\alpha\eta)$ and $q_2|_{\xi=0} = w_2(\eta)\exp(2i\alpha\eta)$, where $w_{1,2}(\eta)$ describe the profiles of lowest-order odd lattice solitons and α is the incident angle. If $\alpha \le \alpha_{\text{cr}}$, the lattice soliton oscillates within a central channel [Fig. 4(a)], but at $\alpha > \alpha_{\text{cr}}$ it moves across the lattice. As a result of this motion the soliton radiates and can be trapped in one of the guiding channels [Fig. 4(b)]. Switching to the desired channel can be achieved within the interval of incident angles that decreases with growth of the output channel number [Fig. 4(c)]. Notice that growth of energy flow at fixed α is accompanied by a decrease of lattice soliton mobility [Fig. 4(d)].

To conclude, we stress that the possibility of packing an arbitrary number of individual lattice solitons into a stable soliton train opens new prospects for managing multihumped structures. Individual solitons that form such a train can be extracted at the entrance of the lattice through the controllable splitting that suggests discrimination in escape angles for solitons that have different positions in the initial train. The concept reported here can be extended to lattice solitons in cubic, photorefractive nonlinear media.

This work was supported by the Generalitat de Catalunya and by the Spanish Government through grant TIC2000-1010. Y. V. Kartashov's e-mail address is azesh@genphys.phys.msu.ru.

*Also with the Department of Physics, M. V. Lomonosov Moscow State University, Moscow, Russia.

†Also with Departamento de Fisica y Matematicas, Universidad de las Americas—Puebla, Cholula, Mexico.

References

1. D. N. Christodoulides and R. I. Joseph, Opt. Lett. **13,** 794 (1988).
2. D. N. Christodoulides, F. Lederer, and Y. Silberberg, Nature **424,** 817 (2003).
3. A. A. Sukhorukov, Y. S. Kivshar, O. Bang, and C. M. Soukoulis, Phys. Rev. E **63,** 016615 (2000).
4. O. Bang, P. L. Christiansen, and C. B. Clausen, Phys. Rev. E **56,** 7257 (1997).
5. T. Peschel, U. Peschel, and F. Lederer, Phys. Rev. E **57,** 1127 (1998).
6. A. Kobyakov, S. Darmanyan, T. Pertsch, and F. Lederer, J. Opt. Soc. Am. B **16,** 1737 (1999).
7. B. A. Malomed, P. G. Kevrekidis, D. J. Frantzeskakis, H. E. Nistazakis, and A. N. Yannacopoulos, Phys. Rev. E **65,** 056606 (2002).
8. T. Pertsch, U. Peschel, and F. Lederer, Opt. Lett. **28,** 102 (2003).
9. A. B. Aceves, C. De Angelis, T. Peschel, R. Muschall, F. Lederer, S. Trillo, and S. Wabnitz, Phys. Rev. E **53,** 1172 (1996).
10. W. Krolikowski and Y. S. Kivshar, J. Opt. Soc. Am. B **13,** 876 (1996).
11. O. Bang and P. D. Miller, Opt. Lett. **21,** 1105 (1996).
12. J. W. Fleischer, T. Carmon, M. Segev, N. K. Efremidis, and D. N. Christodoulides, Phys. Rev. Lett. **90,** 023902 (2003).
13. J. W. Fleischer, M. Segev, N. K. Efremidis, and D. N. Christodoulides, Nature **422,** 147 (2003).
14. D. Neshev, E. Ostrovskaya, Y. Kivshar, and W. Krolikowski, Opt. Lett. **28,** 710 (2003).
15. N. K. Efremidis, J. Hudock, D. N. Christodoulides, J. W. Fleischer, O. Cohen, and M. Segev, Phys. Rev. Lett. **91,** 213906 (2003).
16. Y. V. Kartashov, A. S. Zelenina, L. Torner, and V. A. Vysloukh, Opt. Lett. **29,** 766 (2004).
17. Y. V. Kartashov, L. Torner, and V. A. Vysloukh, Opt. Lett. **29,** 1102 (2004).
18. Y. V. Kartashov, L. Torner, and V. A. Vysloukh, Opt. Lett. **29,** 1117 (2004).
19. D. Mihalache, F. Lederer, D. Mazilu, and L. C. Crasovan, Opt. Eng. **35,** 1616 (1996).