Four-wave mixing of light beams with engineered orbital angular momentum in cold cesium atoms

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We report an experimental demonstration that shows that the spatial structure carried by engineered coherent superpositions of light beams with orbital angular momentum can be mapped into the nonlinear polarization induced in a cloud of cold cesium atoms. The structure of such polarization was revealed by nearly degenerate four-wave-mixing processes. © 2004 Optical Society of America

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The orbital angular momentum (OAM) of light has been the focus of increasing interest during the past few years, since the pioneering work of Allen *et al.*[1] In the paraxial approximation the OAM of light beams, which is associated with their spiral wave fronts, can be separated from the spin angular momentum associated with the polarization of the corresponding electromagnetic field. In the classical domain the OAM of light can be transferred to trapped material particles causing them to rotate (see, e.g., Refs. 2–4 and references therein), a property with important applications in optical tweezing. In the quantum domain, photon pairs generated in spontaneous downconversion are entangled in the OAM, a property that can be employed to generate multidimensional entangled states in arbitrary Hilbert dimensions,[5–8] thus making possible the exploration of deeper quantum features or the proof-of-principle implementation of capacity-increased quantum information schemes.

The OAM content of a light beam can be conveniently represented by a coherent superposition of Laguerre–Gaussian (LG) modes. In many physical settings and applications coherent multimode superpositions are required. For example, in optical tweezing applications, suitable transparent isotropic particles can be set into rotation by suitable multimode beams with zero average OAM.[9] In the domain of quantum optics and its applications, coherent superpositions of LG modes are key ingredients, e.g., in the generation of engineered biphoton entangled states,[10] in the creation of complex atom traps,[11] or in schemes to track the motion of single atoms inside high-finesse optical resonators.[12] For many such quantum-information applications it is important to show that light beams represented by arbitrary multimode coherent superpositions of LG modes can be mapped into the nonlinear polarization induced in clouds of ultracold atoms. The case of pure OAM states was addressed recently.[13–15] It is worth mentioning that the capability of performing quantum-information processing is strongly conditioned to allow reversible storing of a quantum state of light in a long-lived atomic coherence.[16,17] Here we report the first, to our knowledge, experimental demonstration of the mapping of a classical complex LG multimode light beam into the spatial structure of a ground-state coherence induced in an atomic cloud. The experiments were performed in cesium atoms in a four-wave-mixing (FWM) geometry. FWM schemes have been comprehensively investigated in fibers and bulk media (see, e.g., Refs. 18–20 and references therein).

Our aim here is to explore the polarization $\mathcal{P}_C$ induced in a system of cold cesium atoms when three beams $\mathcal{A}_F$, $\mathcal{A}_B$, and $\mathcal{A}_S$ interact with the atomic system. Light beam $\mathcal{A}_S$ is a coherent superposition of light beams that carry OAM, whereas beams $\mathcal{A}_F$ and $\mathcal{A}_B$ are assumed to be Gaussian beams. The polarization induced in the cold atomic sample generates, through a nearly degenerate FWM scheme, a light beam $\mathcal{A}_C$ whose spatial structure reveals the corresponding spatial distribution of polarization $\mathcal{P}_C$ induced in the system of cold atoms.

We generate a noncollinear superposition of LG modes that propagate at slightly different angles around the z axis, $\mathcal{A}_S(z,\mathbf{r}_\perp) = \sum_{n=1}^{N} A_n u_0^n(x,y,z) \times \exp(ik\theta_n x)$, where $\mathcal{A}_S(z,\mathbf{r}_\perp)$ is the cw slowly varying envelope electric field, z is the direction of propagation, $\mathbf{r}_\perp = (x,y)$ is the position in the transverse plane, and θ_n is a small tilt angle with the z axis. LG mode $u_0^l(x,y,z)$ at its beam waist ($z=0$) writes $u_0^l(\rho,\varphi) \propto (\rho/w_0)^{|l|}\exp(-\rho^2/w_0^2)\exp(il\varphi)$, where (ρ,φ) are cylindrical coordinates; w_0 is the beam width; and the index l, referred to as the winding number, describes the helical structure of the wave front around a phase dislocation. Any field distribution can be decomposed into LG modes, i.e., $\mathcal{A}_S(z,\mathbf{r}_\perp) = \sum_{l=-\infty}^{\infty}\sum_{p=0}^{\infty} C_p^l u_p^l(x,y,z)$, where the coefficients $\{C_p^l\}$ determine the OAM content of the light beam[6] when the LG modes are conveniently normalized. The OAM content $\{C_p^l\}$ of light beam $\mathcal{A}_S$ is given by

$$C_p^l = \sum_{n=1}^{N} A_n \int \mathrm{d}x \int \mathrm{d}y u_0^n(x,y,z=0)[u_p^l(x,y,z=0)]^* \times \exp(ik\theta_n x). \quad (1)$$

The specific intensity pattern $|\mathcal{A}_c(x,y)|^2$ detected at a given transverse plane is a signature of the OAM content of the beam. In Fig. 1 we plot the spatial pattern (intensity) to be observed for two specific coherent superpositions of LG beams. Note that the specific pattern to be observed depends on the beam width and radius of curvature of each of the beams at the detection plane.

The experimental setup employed is shown schematically in Fig. 2. We employed the standard FWM configuration, described in detail in Ref. 14. Forward (F) and backward (B) pumping beams are nearly counterpropagating, have the same linear polarization, and are red detuned by 12 MHz with respect to the atomic resonance frequency of the cesium closed transition $6S_{1/2}, F=4-6P_{3/2}, F'=5$. The incident signal beam (S), forming a small angle with the forward beam, has its frequency scannable around that of the pump beams and is orthogonally polarized to them. In such a configuration the nonlinear medium generates a phase conjugate beam (C), which originates exclusively from the induced coherence between pairs of Zeeman sublevels in the ground and excited states. We use a Ti:sapphire laser to magneto-optically trap the atoms and to generate the FWM beams. A pair of acousto-optic modulators (AOMs) is employed to produce a beam with a scannable frequency around that of the pump beams. This beam is directly injected, through the side port of an optical isolator, into a single-mode diode laser, thus locking its frequency and making it highly correlated with that of the pump beams. Different superpositions involving LG modes with topological charges $l=0$, $l=1$, and $l=2$ were generated by use of computer-generated Fresnel-zone plate masks in the arms of the double Mach–Zehnder interferometer arrangement shown in the inset of Fig. 2. Each of the three beam components of the coherent superposition has approximately the same power of 10 μW. However, although the radii of curvature of the beams with $l=1$ and $l=2$ are approximately the same at the output of the interferometer, they are slightly different from that of the Gaussian beam ($l=0$). The selected superposition is then focused into the trap region with a beam waist much smaller than the trap size ($\approx$2 mm). The pump beams, with approximately the same power of 2 mW, are collimated to a diameter of $\approx$5 mm and form an angle of $\theta = 3°$ with signal beam S. We first recorded the spectrum of the FWM-generated signal as a function of the pump-signal detuning δ for a signal beam S corresponding to the different superposition of LG modes. The FWM spectrum, as well as the generated beam profile, were recorded within the $\approx$1-ms time interval during which the trapping beams were blocked by a mechanical chopper. The generated FWM signal associated with a different superposition in the S beam presents a peak around $\delta = 0$ with a subnatural linewidth of the order of 200 kHz, indicating that it originates from a long-lived Zeeman ground-state coherence. Furthermore, we did not observe appreciable saturation of the FWM signal for different combinations of LG beams.

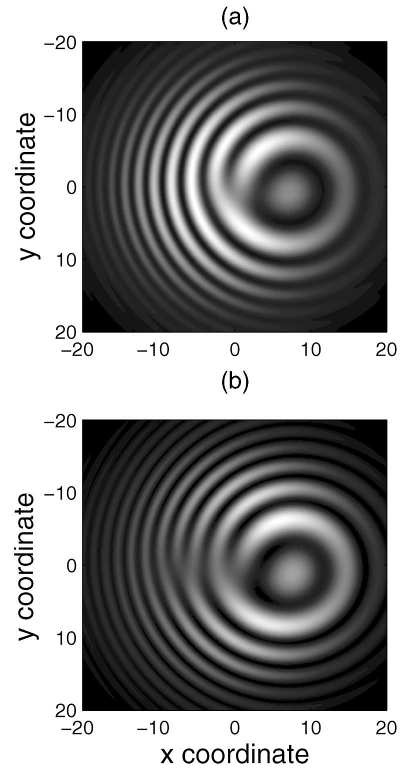

Fig. 1. Normalized spatial light distribution obtained for a noncollinear superposition of LG beams. (a) Superposition of two LG modes with ($l=0, p=0$) and ($l=1, p=0$), with $A_0 = A_1$, $\theta_0 = 0$, $\theta_1 = 0.15$ mrad. (b) Superposition of two LG modes with ($l=0, p=0$), ($l=1, p=0$), and ($l=2, p=0$), with $A_0 = A_1 = A_2$, $\theta_0 = 0$ mrad, $\theta_1 = \theta_2 = 0.15$ mrad. In all cases the beam width of the Gaussian beam ($l=0$) is $w_0 = 20$ mm and the radius of curvature is $R_0 = 5 \times 10^4$ m. For all beams with $l \neq 0$ the beam width is $w_l = 10$ mm and $R_l = 1 \times 10^9$. All distances are in millimeters.

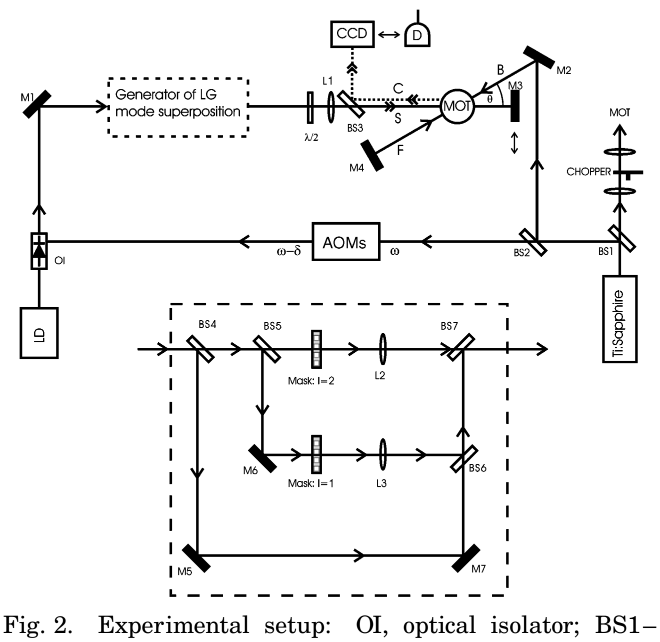

Fig. 2. Experimental setup: OI, optical isolator; BS1–BS7, beam splitters; M1–M7, mirrors; L1–L3, lenses; MOT, magneto-optic trap; $\lambda/2$, half-wave plate; D, detector; LD, laser diode. Other terms defined in text. Inset, double Mach–Zehnder interferometer to generate different superpositions of LG modes.

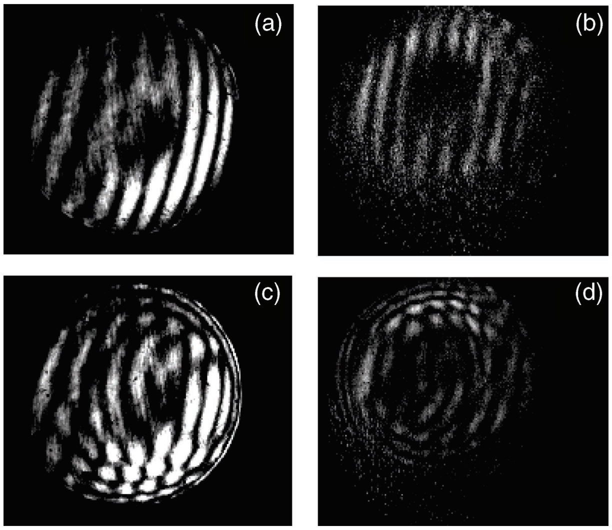

Fig. 3. (a) Incident light spatial distribution pattern corresponding to the noncollinear superposition of two LG beams with $l = 1$ and $l = 2$. (b) Spatial profile of the generated superposition corresponding to the beam observed in (a). (c) Incident light spatial distribution pattern corresponding to the noncollinear superposition of three LG beams with $l = 0$, $l = 1$, and $l = 2$. (d) Spatial profile of the generated superposition corresponding to the incident beam described in (c). The presented images are obtained by subtracting from the corresponding image the dark image, recorded when the injection of the diode laser is blocked, where no coherent signal is observed.

The spatial profile of the generated superposition is analyzed by a CCD camera. Retroreflecting the incident superposition with auxiliary removable mirror M4 allowed us to record its spatial profile also. In Fig. 3(a) we show the spatial profiles (intensity) for an incident superposition corresponding to $l = 1$ and $l = 2$ beams. The spatial profile of the corresponding generated superposition is shown in Fig. 3(b) and, as can be seen, presents the same fork structure. In fact, with the help of the Gaussian beam ($l = 0$) we verified that the components of the generated superposition indeed consist of single beams with $l = 1$ and $l = 2$. The fact that the fork structure is reversed in the previous patterns can be understood as being due to a change of sign in the angle between the interfering beams associated with the incident and generated superpositions.[21] We observed similar results for the case of superpositions involving $l = 1$ and $l = 2$ with $l = 0$. Furthermore, in Fig. 3(c) we show the intensity distribution for an incident superposition containing the $l = 0$, $l = 1$, and $l = 2$ LG modes, and in Fig. 3(d) we show the corresponding generated superposition, and one can clearly see that the generated mode also preserves the initial superposition. In these last patterns, components $l = 1$ and $l = 2$ are misaligned as in the previous case, but component $l = 0$ is made collinear with component $l = 2$.

In conclusion, we have experimentally demonstrated the generation of different coherent superpositions of light beams carrying OAM by use of a FWM scheme by means of an induced coherence grating in the Zeeman ground-state sublevels of cold cesium atoms. The induced coherence was revealed to contain the spatial phase structure associated with the incident superposition of LG modes. This experiment demonstrated that the information encoded in the OAM content of a light beam can be transferred, through interaction with an atomic system by means of a FWM process, to another light beam mode. This is an important tool for multidimensional information processing, owing to the possibility of manipulating quantum information stored in long-lived atomic coherence.

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