Stable soliton complexes in two-dimensional photonic lattices

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We show that two-dimensional photonic Kerr nonlinear lattices can support stable soliton complexes composed of several solitons packed together with appropriately engineered phases. This may open up new prospects for encoding pixellike images made of robust discrete or lattice solitons. © 2004 Optical Society of America

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A periodic transverse modulation of the refractive index can profoundly affect the properties of optical spatial solitons formed in a nonlinear medium through the competition of diffraction and nonlinear self-action effects. In such periodic systems, discrete solitons are known to be possible in the waveguiding sites of the lattice.[1] This class of self-trapped states exhibits several unique features, among them angle- and power-controlled steering that can be effectively used in all-optical switching and routing architectures.[2] The advantages stemming from the transverse periodic variation of the refractive index of a nonlinear medium are clearly visible in harmonic nonlinear lattices (i.e., involving two-dimensional harmonic potentials). For instance, lattices of this kind can be used to implement systems with tunable discreteness since they are capable of operating in regimes of both weak and strong coupling between neighboring sites depending on the depth and period of refractive-index modulation.[3,4]

Recently lattice solitons were predicted and experimentally observed in photorefractive crystals in both one and two transverse dimensions.[5–9] Harmonic lattices in such crystals can be induced all-optically with several interfering plane waves whose intensity and intersection angles define the lattice depth and period.[7] We have recently shown that besides lowest-order solitons, one-dimensional harmonic lattices with a saturable Kerr-type nonlinearity can support stable soliton combinations or lattice soliton trains, a phenomenon that opens up new prospects for observing and managing multihumped structures.[10] Even though the basic properties[5–9] and stability[11] of lowest-order solitons in two-dimensional lattices are by now well established, the existence of more general soliton states or soliton complexes is still an open problem.

In this Letter we study the general properties and stability of complex families of two-dimensional lattice solitons in cubic Kerr-type optical media. We find that nontrivial lattice soliton complexes or matrices can exist that are analogous to twisted strongly localized modes of two-dimensional discrete systems. Such soliton complexes are higher-order, stationary solutions of the governing equations of light propagation in photonic lattices, and we find them to be completely stable above a certain power threshold. Intuitively, the complexes can be viewed as nonlinear superposition of several lowest-order odd solitons with appropriately engineered phases.

Consider a nonlinear Kerr medium with a periodic modulation of the linear refractive index in the transverse direction. In this case the dimensionless amplitude q of a laser beam propagating along the z axis of this material is described by the following nonlinear evolution equation:

$$i\,\frac{\partial q}{\partial \xi} = -\frac{1}{2}\left(\frac{\partial^2 q}{\partial \eta^2} + \frac{\partial^2 q}{\partial \zeta^2}\right) + \sigma q|q|^2 - pR(\eta,\zeta)q\,, \tag{1}$$

where η, ζ, and ξ represent the transverse and longitudinal coordinates scaled to the characteristic beam width and diffraction length, respectively. Parameter σ takes values of $\sigma=-1(+1)$ for focusing (defocusing) media, and p is the guiding parameter that is proportional to the linear refractive-index modulation depth. Function $R(\eta,\zeta) = \cos(2\pi\eta/T)\cos(2\pi\zeta/T)$ describes the transverse periodic refractive-index profile with spatial period T. Equation (1) admits several conserved quantities including total energy flow $U = \int_{-\infty}^{\infty}|q|^2 \mathrm{d}\eta \mathrm{d}\zeta$.

We search for the trivial-phase stationary solutions of Eq. (1) with the form $q(\eta,\zeta,\xi) = w(\eta,\zeta)\exp(ib\xi)$, where $w(\eta,\zeta)$ is a real function and b is a real propagation constant. Mathematically, families of two-dimensional lattice solitons are defined by propagation constant b, modulation period T, and guiding parameter p. Since scaling transformation $q(\eta,\zeta,\xi,p) \rightarrow \chi q(\chi\eta,\chi\zeta,\chi^2\xi,\chi^2 p)$ can be used to obtain various families of lattice solitons from a given family, we selected the transverse scale in such a way that modulation period $T=\pi/2$, and we varied b and p. Linear stability analysis of the lattice solitons obtained was carried out by searching for the perturbed solutions of Eq. (1) in the form $q(\eta,\zeta,\xi) = [w(\eta,\zeta) + u(\eta,\zeta,\xi) + iv(\eta,\zeta,\xi)]\exp(ib\xi)$, with u and v being the real and the imaginary parts of the perturbation, respectively, which can grow with complex growth rates δ. A standard linearization

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procedure for Eq. (1) yields coupled Schrödinger-type equations for perturbation components u and v that can be solved numerically with a split-step Fourier method to give the perturbation profile and corresponding growth rate.

Here we concentrated our attention mainly on the self-focusing case; thus we set $\sigma = -1$. Two-dimensional optical lattices support two types of the lowest-order solitons—single-hump and two-hump lattice solitons—whose properties are summarized in Fig. 1. We note that the maximum intensity of a single-hump discrete soliton resides on only one waveguiding site of function $pR(\eta, \zeta)$ [Fig. 1(a)], whereas in the case of a two-hump soliton the two equal intensity maxima are located at two neighboring lattice sites. Thus two-hump lattice solitons can be intuitively viewed as a nonlinear combination of two in-phase single-hump states [Fig. 1(b)]. Because of the lattice structure, the minimum separation between single-hump solitons can be T (if the line connecting the soliton centers is parallel to the η or ζ axis) or $T/2^{1/2}$ [if the line connecting the soliton centers is at a 45° angle with respect to the η and ζ axes as shown in Fig. 1(b)]. Here we consider the latter case because it corresponds to a tighter packing of elementary solitons or soliton bits. The energy flow associated with both single- and two-hump solitons is found to be a nonmonotonic function of the propagation constant, as is clearly seen from Fig. 1(c). Figure 1(c) also demonstrates that two approximately equal propagation constant cutoffs b_{co} exist where the energy flow diverges as $b \to b_{\text{co}}$. In the region slightly below cutoff the soliton transforms into a linear Bloch wave. Using linear stability analysis, we found that two-dimensional single-hump lattice solitons are unstable in the narrow region near cutoff where $\mathrm{d}U/\mathrm{d}b \le 0$ and become stable in the region where $\mathrm{d}U/\mathrm{d}b > 0$, which is in agreement with the Vakhitov–Kolokolov stability criterion for ground soliton states [inset in Fig. 1(d)]. Conversely, the two-hump discrete soliton is exponentially unstable in the entire domain of its existence [Fig. 1(d)].

A qualitatively different situation emerges when two lattice solitons involved in the complexes are out of phase [Fig. 2(a)]. We found stationary field distributions $w(\eta, \zeta)$ in the form of such complexes by solving Eq. (1) with the substitution $q(\eta, \zeta, \xi) = w(\eta, \zeta)\exp(ib\xi)$. Such a composite structure is somehow equivalent to a twisted localized mode in a discrete system.[12] As in the case of single and two-hump solitons, the energy flow is a nonmonotonic function of the propagation constant [Fig. 2(b)]. The propagation constant has a lower cutoff, but the energy flow does not diverge as $b \to b_{\text{co}}$ [Fig. 2(b)]. In addition, cutoff b_{co} is a nonmonotonic function of guiding parameter p, and it goes to infinity for both shallow $p \to 0$ and deep $p \to \infty$ lattices [Fig. 2(c)]. One of the central results of our work is that such stationary twisted, higher-order modes in two-dimensional lattices become completely stable above a certain threshold energy level or, equivalently, above a critical propagation constant value b_{cr} [Fig. 2(d)]. Note that at the energy level corresponding to the stabilization threshold, the spots forming the twisted solitons strongly overlap each other, thus the repulsive interaction acting between them without the lattice would be large. However, such forces are fully compensated by the periodic potential introduced by the lattice; thus stable propagation does occur. In the presence of broadband random perturbations, twisted modes propagate undistorted over thousands of diffraction lengths. Note that close to the lower cutoff, twisted modes become oscillatory unstable. The corresponding growth rate has small real and large imaginary parts [Fig. 2(d)]. We have found that an increase in the refractive-index modulation

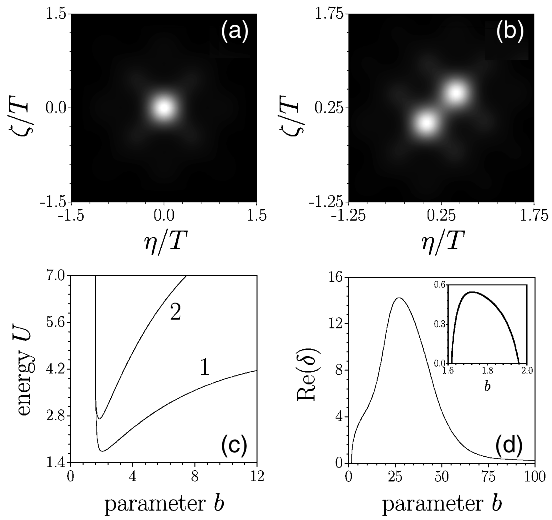

Fig. 1. Profiles of (a) single-hump and (b) two-hump lattice solitons at $b = 2.5$. (c) Energy flow versus propagation constant for single- (curve 1) and two-hump (curve 2) solitons. (d) Real part of the perturbation growth rate for single- (inset) and two-hump lattice solitons versus propagation constant. Guiding parameter $p = 10$.

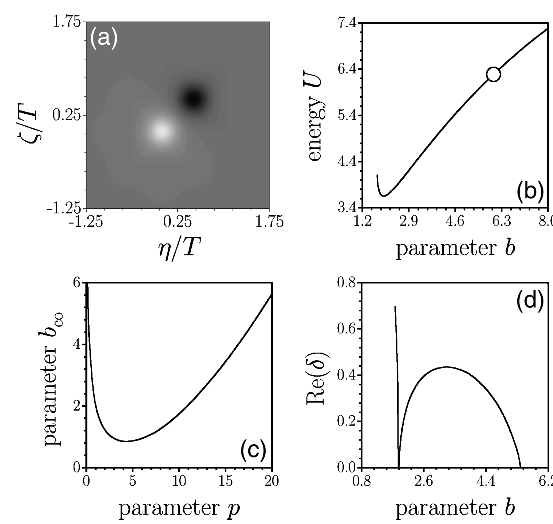

Fig. 2. (a) Profile of a stable twisted mode corresponding to a point marked by a circle in dispersion diagram (b). For (a) and (b), guiding parameter $p = 10$. (c) Cutoff on propagation constant versus guiding parameter. (d) Real part of perturbation growth rate versus propagation constant at $p = 10$.

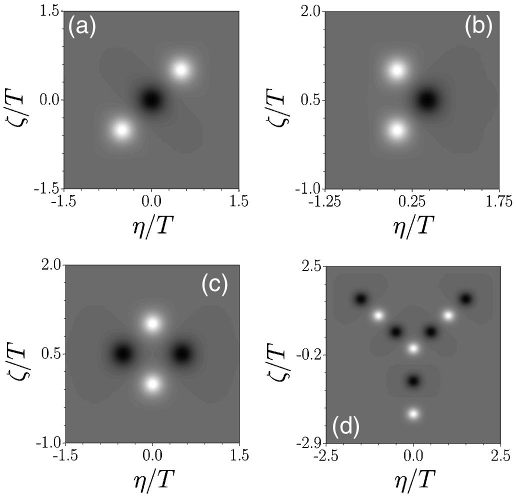

Fig. 3. Simplest stable combinations of lattice solitons at $p = 10$ corresponding to (a), (b) $b = 8$ and (c) $b = 11$. Example of stable soliton letter Y composed from nine lowest-order lattice solitons at $b = 16$ with $p = 20$.

depth leads to a decrease in the difference $b_{\text{cr}} - b_{\text{co}}$ (the width of instability area).

Even more complicated soliton complexes become stabilized above a certain energy flow level. Examples of such stable complexes that were found by solving Eq. (1) are shown in Figs. 3(a)–3(c). Note that the neighboring lattice solitons in stable matrices are π out of phase. The properties of these soliton complexes, including their $U(b)$ and $b_{\text{co}}(p)$ characteristics, are qualitatively similar to those of the twisted modes in Fig. 2. We found that the higher the number of solitons involved in the complexes, the higher the energy threshold for stabilization and that such a threshold grows monotonically with the number of solitons. The critical propagation constant at which stabilization occurs also increases with the number of solitons contained in the complexes. The matrices depicted in Figs. 2(a) and 3(a)–3(c) can be viewed as elementary building blocks for soliton letters and images that can be transferred in a stable way along the lattice even in the presence of noise. An example of a stable capital soliton letter Y is shown in Fig. 3(d). Another important issue has to do with the excitation of such stable soliton matrices. Input field distributions necessary for the formation of such arrays can be obtained with amplitude–phase masks. Note that different spots forming the soliton complexes carry the overall phases fixed by the twisted soliton solutions; thus they do not exist when phases of neighboring spots take arbitrary values.

Defocusing optical lattices can also support bright solitons.[9,13,14] We have found that, in contrast with the case of focusing lattices, in defocusing systems the lattice soliton matrices are stable when all solitons are in phase, whereas twisted modes are exponentially unstable in the entire domain of their existence. In defocusing media a narrow instability band for soliton complexes occurs only near an upper cutoff of the propagation constant.

In conclusion, we have analyzed the properties of two-dimensional lattice solitons in Kerr-type nonlinear media with a harmonic transverse modulation of the refractive index. We have discovered that besides the simplest odd solitons such lattices can support stationary, stable soliton complexes, which can be intuitively viewed as nonlinear superposition of several odd solitons. This result suggests the possibility of packing an arbitrary number of individual solitons into one stable matrix to encode complex soliton images and letters.

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