Soliton spiraling in optically induced rotating Bessel lattices

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Received July 19, 2004

We address soliton spiraling in optical lattices induced by multiple coherent Bessel beams and show that the dynamic nature of such lattices makes it possible for them to drag different soliton structures, setting them into rotation. We can control the rotation rate by varying the topological charges of lattice-inducing Bessel beams. © 2005 Optical Society of America

OCIS codes: 190.5530, 190.4360, 060.1810.

Propagation of light in media with modulated refractive indices exhibits a variety of unique phenomena. When the modulation forms an array of evanescently coupled waveguides, discrete solitons appear as excitations of a few guides.[1] Discrete solitons can move across the array, a phenomenon accompanied by radiation but with important all-optical routing applications. Tuning the depth of the refractive-index modulation is a powerful tool that allows one to control soliton mobility.[2] Such tunable arrays or lattices can be generated optically in photorefractive media.[3–6]

To date, soliton mobility has been analyzed in lattices whose shapes remain invariant along the longitudinal direction. In this Letter we explore, for the first time to our knowledge, optically induced lattices whose shapes evolve dynamically. In particular, we address a phenomenon of soliton spiraling in a lattice that drags solitons into rotary motion. Such lattices can be generated by the interference of several Bessel beams.[7] The resultant refractive-index modulation produces guiding structures that resemble optical fibers with varying shapes. However, the possibility of reconfiguring and optically tuning the properties of such structures is a central point of the concept described here. We discuss the effects of lattice strength on soliton motion as well as the possibility of controlling the soliton rotation rate by changing topological charges of lattice-creating beams.

We consider propagation of light along the z axis of a focusing cubic medium with transverse modulation of its refractive index, described by the nonlinear Schrödinger equation for dimensionless complex field amplitude q:

$$i\frac{\partial q}{\partial \xi} = -\frac{1}{2}\left(\frac{\partial^2 q}{\partial \eta^2} + \frac{\partial^2 q}{\partial \zeta^2}\right) - q|q|^2 - pR(\eta,\zeta,\xi)q. \qquad (1)$$

Here the longitudinal (ξ) and transverse (η and ζ) coordinates are scaled to the diffraction length and the input-beam width, respectively; parameter p is proportional to the refractive-index modulation depth; the function $R(\eta,\zeta,\xi)$ describes the profile of the lattice. Among the conserved quantities of Eq. (1) is the energy flow $U=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|q|^2 \mathrm{d}\eta\,\mathrm{d}\zeta$. We address refractive-index modulations induced optically with interfering Bessel beams $J_k[(2b_k)^{1/2}r]\exp(ik\phi - ib_k\xi)$ with various topological charges k. Here $r^2=\eta^2+\zeta^2$ is the radius, ϕ is the azimuth angle, and the parameter b_k defines the transverse scale of the kth beam. Good approximations of Bessel beams can be created experimentally by a variety of techniques.[8,9] We assume that the lattice shape exhibits the intensity of the interference pattern, $R(\eta,\zeta,\xi) \sim \Sigma_k J_k^2 + 2\Sigma_n\Sigma_{m>n} \times J_nJ_m \cos[(n-m)\phi + (b_m-b_n)\xi]$ of several beams. In the simplest case of two Bessel beams with b_n and b_m chosen such that both functions $J_n[(2b_n)^{1/2}r]$ and $J_m[(2b_m)^{1/2}r]$ acquire their maximal values at $r=1$, one gets a twisted lattice with a rotation rate defined by b_m-b_n and an azimuthal symmetry defined by the value of $n-m$. Except for the rotation, the cross section of such a lattice is invariant along ξ. There are two characteristic longitudinal scales for the twisted lattices: total rotation period $T_{\mathrm{rot}}=2\pi(m-n)/(b_m-b_n)$ and distance $T=2\pi/(b_m-b_n)$, where the lattice shape replicates itself. For the simplest lattice with $m-n=1$ one has $T_{\mathrm{rot}}=T$. Even though the induced refractive-index modulation decreases as $r\to\infty$, we use the term "optical lattice" to stress the possibility of reconfiguring it all-optically. Note that the infinite extent of harmonic lattices is important only in linear propagation, when many lattice sites are occupied, in contrast to the nonlinear case addressed here.

To study soliton dragging we fixed the charge $n=1$ and varied m. For $m=2$ the twisted lattice has one clearly pronounced guiding channel [Fig. 1(a)]; in that case we set the input field distribution in the form of a single soliton supported by the lattice at $\xi=0$. Such a beam would propagate undistorted in the nonrotating lattice. The soliton beam is characterized by its energy flow U. Narrow solitons cease to feel the lattice, and their energy flow approaches that of Townes solitons: $U_{\max}=5.85$. In the twisted dynamic lattices such beams are no longer stationary, but for deep enough lattices the beams can be set into steady spiral motion with a period dictated by the period of the lattice rotation (Fig. 1). Such soliton motion is conceptually different from that described in previous papers on soliton steering and spiraling,[10,11] as here the otherwise immobile solitons are dragged by the dynamic lattice. We found that solitons can fol-

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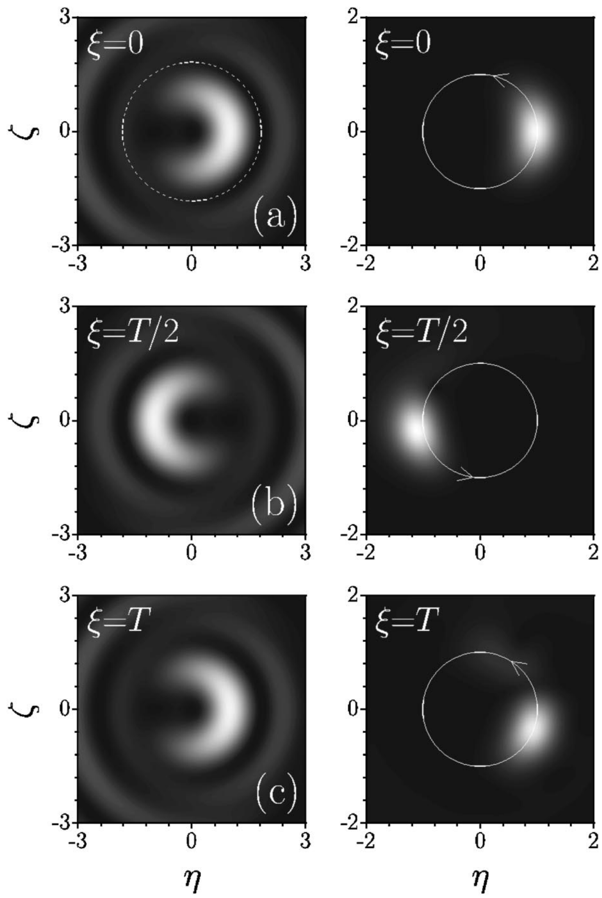

Fig. 1. Soliton propagation in a lattice with $m=2$. Refractive index (left) and field (right) distributions at various distances are shown. White circles at the right display trajectories of the lattice maximum, and arrows show the rotation direction. Input energy flow, $U_{\text{in}}=1.5$; $p=32$.

low the lattice for huge distances, exceeding any feasible crystal length, and can undergo thousands of rotations. The location of the soliton peak amplitude exhibits small oscillations inside a rotating guiding channel of the lattice (Fig. 1). In the case of a lattice-creating beam with charge $m=3$ (Fig. 2), two pronounced guiding channels can be used to drag more-complicated soliton structures such as dipole-mode solitons. We have found the input profile of a dipole-mode soliton as an exact solution of Eq. (1) at $\xi=0$. The limiting energy flow for the existence of dipole-mode solitons is given by $2U_{\max}$, and close to this value the dipole-mode solitons are transformed into two narrow and almost noninteracting solitons. Dipole-mode solitons are also set into rotary motion by the lattice (Fig. 2). Besides a dipole-mode soliton, the lattice shown in Fig. 2 can drag a single soliton located in either of two guiding sites.

To show that nonlinearity is essential for steady soliton spiraling, we compared propagation of identical beams in the lattice shown in Fig. 1 with and without nonlinearity [Figs. 3(a) and 3(b)]. In the linear regime, radiation losses grow dramatically and the beam broadens and is redistributed between secondary lattice rings, thus quickly spreading after several lattice rotations. In the nonlinear regime the radiation is orders of magnitude smaller. To quantify the radiation rate we calculated energy flow U_ξ concentrated in close proximity to the main guiding lattice sites [within the ring shown by dashed circles in Figs. 1(a) and 2(a)] for different propagation dis-

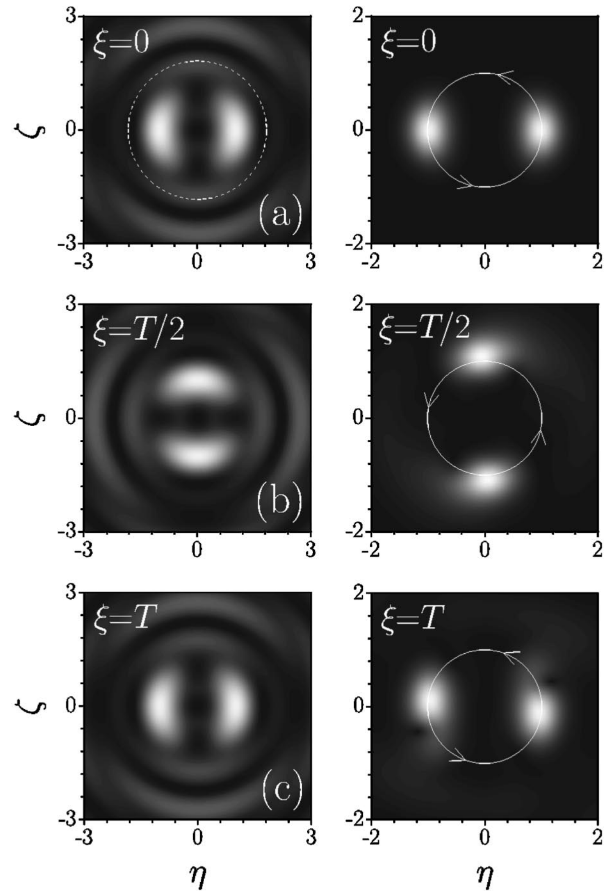

Fig. 2. The same as in Fig. 1 but for a dipole-mode soliton in the lattice with $m=3$ at $U_{\text{in}}=1.5$ and $p=50$.

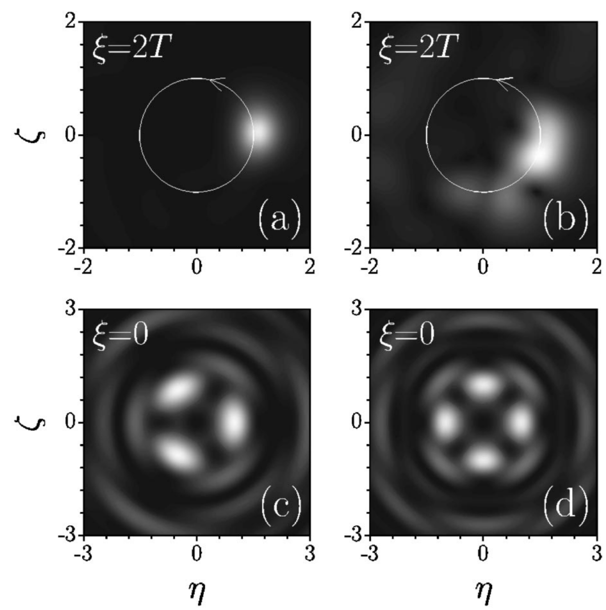

Fig. 3. Output field distributions after two rotations of the lattice with $m=2$ in (a) nonlinear and (b) linear media at $U_{\text{in}}=4$ and $p=25$. Lattices with (c) $m=4$ and (d) $m=5$.

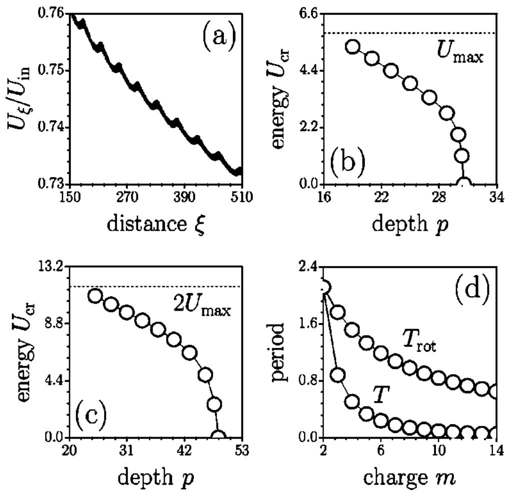

Fig. 4. (a) Energy flow of a single soliton captured by the lattice with $m=2$ versus propagation distance at $U_{in}=1.5$ and $p=32$. (b) Critical energy versus lattice depth for a single soliton captured by the lattice with $m=2$ and (c) dipole-mode soliton captured by the lattice with $m=3$. (d) Rotation T_{rot} and self-replication T periods versus charge m.

tances [Fig. 4(a)]. The radiation rate is larger during several initial rotations, but it is reduced drastically with increasing ξ, and beyond $\xi=150$ the total radiative losses over a hundred diffraction lengths (~50 rotations) amount to ~1%. A similar result was obtained for dipole-mode solitons dragged by the lattice shown in Fig. 2.

We also found that the radiation rate decreases with growth of the lattice depth and of the input soliton energy flow. To quantify this effect we set the criterion that the soliton is captured by the lattice when radiative losses after one rotation (for a dipole-mode soliton after one self-replication period) amount to less than 5%. Inasmuch as losses decrease monotonically with increasing input energy flow, this phenomenon enables us to define critical energy level U_{cr}. The critical energy decreases with growth of the lattice depth [Figs. 4(b) and 4(c)]. This implies that sufficiently deep lattices become totally trapping and can drag even low-energy beams. However, here we are interested only in the parameter range where nonlinearity really affects the dragging properties of the lattice, as shown in Figs. 3(a) and 3(b). Note that typically at $U\approx U_{cr}$ the nonlinearity's strength is comparable with that of the lattice. With decreasing lattice depth, U_{cr} approaches U_{max} for single solitons and $2U_{max}$ for dipole-mode solitons. This implies that, below a certain critical value of p, the lattice may be unable to trap and drag solitons. This critical value (~18 for the rapidly rotating lattices considered here) is reduced considerably for lattices with lower rotation rates formed by broader Bessel beams. The maximal rotation rate of a twisted lattice is dictated by the minimal achievable size of the core of a lattice-creating beam, which is of the order of a wavelength. Lattices created with such beams are expected to perform tens of rotations per millimeter.

A key result is that, by changing the topological charge of one of the lattice-creating beams, it is possible to control the rotation rate of the lattice and, hence, the output soliton position at a given propagation length [Fig. 4(d)]. Besides changing the rotation rate, an increase of topological charge m results in more-complex refractive-index distributions [Figs. 3(c) and 3(d)]. Such lattices can trap and drag soliton complexes composed from several bright spots as well as single beams located in either of the guiding channels of the structure.

In conclusion, the results reported here introduce an important new concept for soliton control. The new scheme is based on the dragging of different soliton structures that is caused by dynamically varying optical lattices made by multiple interfering Bessel beams. The approach has several control parameters, including the depth of the lattice and the topological charge of the lattice-creating beams.

This work was partially supported by the Government of Spain through grant BFM 2002-2861 and by the Ramon-y-Cajal program. Y. V. Kartashov's e-mail address is yaroslav.kartashov@upc.es.

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