Upper threshold for stability of multipole-mode solitons in nonlocal nonlinear media

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We address the stability of multipole-mode solitons in nonlocal Kerr-type nonlinear media. Such solitons comprise several out-of-phase peaks packed together by the forces acting between them. We discover that dipole-, triple-, and quadrupole-mode solitons can be made stable, whereas all higher-order soliton bound states are unstable. © 2005 Optical Society of America

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The interactions that arise between optical solitons generate a variety of phenomena. Unlike the interactions of scalar solitons, which tend to repel or attract each other depending on their relative phase difference only,[1] the interaction between solitons, which incorporates several field components, may be more complex. Thus the formation of vector multipole-mode solitons is possible in local saturable[2,3] and quadratic[4–6] media. The properties and interactions of solitons are also strongly affected by a nonlocality of the nonlinear response. Nonlocality is typical for photorefractive[1,2,7] and liquid [8,9] crystals; it is characteristic for thermal self-actions[10] and can occur in plasmas.[11] Nonlocality suppresses modulational instability of plane waves,[12,13] and it can arrest collapse and instabilities of two-dimensional and vortex solitons (for a recent review see, e.g., Ref. 14). Nonlocality also affects soliton interactions[8,15] and allows soliton bound states to be formed.[6,15–17] Dipole-mode bright solitons were observed by Hutsebaut *et al.*,[18] whereas the attraction of dark solitons was observed by Dreischuh *et al.*[19] However, the important issue of stability of bound states of bright solitons in nonlocal media has not been addressed. In particular, an open question is: How many solitons can be packed into a stable bound state? In this Letter we report the outcome of such a stability analysis.

We consider the propagation of a slit laser beam along the ξ axis in media with a nonlocal focusing Kerr-type nonlinearity described by the system of phenomenological equations for dimensionless complex light field amplitude q and nonlinear correction to the refractive index n:

$$i\frac{\partial q}{\partial \xi} = -\frac{1}{2}\frac{\partial^2 q}{\partial \eta^2} - qn,$$

$$n - d\frac{\partial^2 n}{\partial \eta^2} = |q|^2, \tag{1}$$

where η and ξ stand for the transverse and the longitudinal coordinates scaled to the beam width and the diffraction length, respectively, and parameter d stands for the degree of nonlocality of the nonlinear response. When $d \to 0$, Eqs. (1) are reduced to a single nonlinear Schrödinger equation; $d \to \infty$ corresponds to a strongly nonlocal regime. Equations (1) describe the nonlinear response of liquid crystals in steady state.[8,9] We neglect transient effects, assuming continuous wave illumination (see Ref. 20 for a recent discussion of the reorientational relaxation time in typical crystals). Equations (1) conserve the energy flow $U=\int_{-\infty}^{\infty}|q|^2 \mathrm{d}\eta$ and the Hamiltonian

$$H = \int_{-\infty}^{\infty}\left[\frac{1}{2}\left|\frac{\partial q}{\partial \eta}\right|^2 - \frac{1}{2}|q|^2\int_{-\infty}^{\infty} G(\eta-\lambda)|q(\lambda)|^2 \mathrm{d}\lambda\right]\mathrm{d}\eta,$$

where $G(\eta)=(1/2d^{1/2})\exp(-|\eta|/d^{1/2})$ is the response function of the nonlocal medium.

We search for stationary soliton solutions of Eqs. (1) numerically in the form $q(\eta,\xi)=w(\eta)\exp(ib\xi)$, where $w(\eta)$ is the real function and b is a propagation constant. To elucidate the linear stability of soliton families we searched for perturbed solutions in the form $q(\eta,\xi)=[w(\eta)+u(\eta,\xi)+iv(\eta,\xi)]\exp(ib\xi)$, where real $[u(\eta,\xi)]$ and imaginary $[v(\eta,\xi)]$ parts of perturbation can grow with a complex rate δ on propagation. Linearization of Eqs. (1) around stationary solution $w(\eta)$ yields the eigenvalue problem

$$\delta u = -\frac{1}{2}\frac{\mathrm{d}^2 v}{\mathrm{d}\eta^2} + bv - nv,$$

$$\delta v = \frac{1}{2}\frac{\mathrm{d}^2 u}{\mathrm{d}\eta^2} - bu + nu + w\Delta n, \tag{2}$$

where $\Delta n = 2\int_{-\infty}^{\infty} G(\eta-\lambda)w(\lambda)u(\lambda)\mathrm{d}\lambda$ is the refractive-index perturbation. We have solved the system of Eqs. (2) numerically.

First we recall the properties of ground-state solitons (Fig. 1). The width of a ground-state soliton increases while its peak amplitude decreases with increasing degree of nonlocality d at fixed U. Energy flow U is a monotonically growing function of b [Fig. 1(b)]. As $b \to 0$ the soliton broadens drastically while its energy flow vanishes. Ground-state solitons are stable in the entire domain of their existence and achieve the absolute minimum of Hamiltonian H for a fixed energy flow U [Fig. 1(c)].

The central result of this Letter is that several types of multipole-mode soliton can also be made completely stable in nonlocal media. Intuitively,

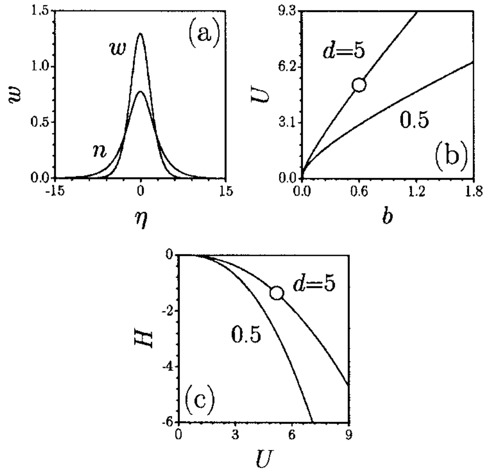

Fig. 1. (a) Profile of a ground-state soliton that corresponds to the points marked by circles in (b) the dispersion diagram and (c) the Hamiltonian-energy diagram.

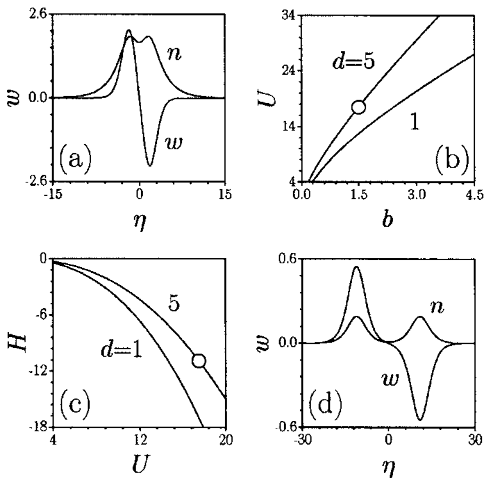

Fig. 2. (a) Profile of a dipole-mode soliton that corresponds to the points marked by circles in (b) the dispersion diagram and (c) the Hamiltonian-energy diagram. (d) Profile of a low-energy dipole-mode soliton that corresponds to $b=0.13$ at $d=5$.

multipole-mode solitons can be viewed as nonlinear combinations (bound states) of fundamental solitons with alternating phases. Such bound states cannot exist in a local Kerr-type medium, in which a π phase difference between solitons causes a local decrease of refractive index in the overlap region and results in repulsion. By contrast, in nonlocal media the refractive-index change in the overlap region depends on the whole intensity distribution in the transverse direction, and under appropriate conditions the nonlocality can lead to an increase in refractive index and to attraction between solitons. The proper choice of separation between solitons results in bound-state formation. Properties of the simplest bound states of two solitons are summarized in Fig. 2. One can see that the refractive-index distribution features a small dip near point $\eta=0$, where the light field vanishes [Fig. 2(a)]. This dip is more pronounced at a small degree of nonlocality, whereas at $d \gg 1$ the refractive-index distribution becomes almost bell shaped. The energy flow of such solitons increases monotonically with increasing b [Fig. 2(b)]. At small energy flows, dipole-mode solitons are transformed into two well-separated out-of-phase solitons, whose amplitudes decrease as b decreases [Fig. 2(d)]. The important result is that dipole-mode solitons are stable in the entire domain of their existence, even for small degrees of nonlocality $d \sim 0.1$ and at low energy levels, when solitons forming a bound state are well separated [Fig. 2(d)].

Note that bound soliton states were also studied in quadratic media, which can be regarded as nonlocal under appropriate conditions and can lead to similar equations for profiles of stationary solitons.[6] However, the principal difference between the two systems becomes apparent in a stability analysis, which results in different eigenvalue problems and, hence, in completely different stability properties of bound soliton states.

To answer the important question about the maximal number of solitons that can be incorporated into a stable bound state we performed stability analysis of a number of higher-order soliton solutions. The results of the stability analysis are summarized in Fig. 3. The energy flow of such solitons also grows monotonically with increasing b. In all cases in the regime of strong nonlocality ($d \gg 1$) the refractive-index distribution for multipole-mode solitons features a bell-shaped profile with a small modulation on its top in accordance with the number of peaks in the soliton

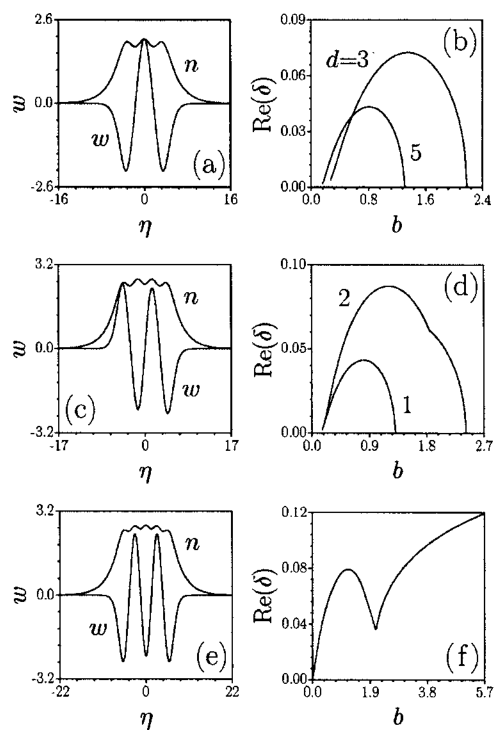

Fig. 3. (a) Profile of a triple-mode soliton at $b=1.5$ and $d=5$. (b) Real part of the perturbation growth rate for the triple-mode soliton versus a propagation constant. (c) Profile of a quadrupole-mode soliton at $b=2$ and $d=5$. (d) Real part of the perturbation growth rate for 1, triple- and 2, quadrupole-mode solitons versus propagation constant at $d=5$. (e) Profile of a fifth-order soliton at $b=2$ and $d=8$. (f) Real part of the perturbation growth rate for the fifth-order soliton at $d=8$.

[Figs. 3(a) and 3(c)]. Stability analysis revealed that low-energy triple- and quadrupole-mode solitons are oscillatory unstable [Figs. 3(b) and 3(d)], but their complete stabilization is possible when the soliton energy flow exceeds a certain threshold. The width of the instability domain as well as the maximum growth rate decreases with increasing degree of nonlocality for both triple- and quadrupole-mode solitons [see, for example, Fig. 3(b)]. It should be pointed out that at fixed d the width of the instability domain for a triple-mode soliton is narrower than that for a quadrupole-mode soliton [Fig. 3(d)]. One of our most important findings is that bound states incorporating five or more solitons were all oscillatory unstable with the framework of model equation (1) [see Figs. 3(e) and 3(f) for a typical profile and dependence Re $\delta(b)$ of an unstable fifth-order soliton]. We found this by performing linear stability analyses for bound states of as many as 12 solitons and d values from the interval (0, 100). In all cases the growth rate for unstable bound states was found to increase as $b \to \infty$, similarly to Fig. 3(f).

To confirm the results of linear stability analysis, we performed numerical simulations of Eqs. (1) with input conditions $q(\eta,\xi=0)=w(\eta)[1+\rho(\eta)]$, where $w(\eta)$ is the profile of the stationary wave and $\rho(\eta)$ is a random function with a Gaussian distribution and variance σ_{noise}^2. Stable dipole- and triple-mode solitons survive over huge distances in the presence of quite considerable broadband input noise (Fig. 4), whereas higher-order solitons self-destroy on propagation.

We also found that the stability of bound soliton states is defined to a great extent by the character of the nonlocal nonlinear response. Thus, in contrast to materials with an exponential response function $G(\eta)=(1/2d^{1/2})\exp(-|\eta|/d^{1/2})$ produced by Eqs. (1) (as in liquid crystals), materials with a Gaussian response function $G(\eta)=(\pi d)^{-1/2}\exp(-\eta^2/d)$ admit of no upper threshold for the number of solitons that can be incorporated into stable bound states. Such a difference makes the search for materials with different characteristics of nonlocal response especially important.

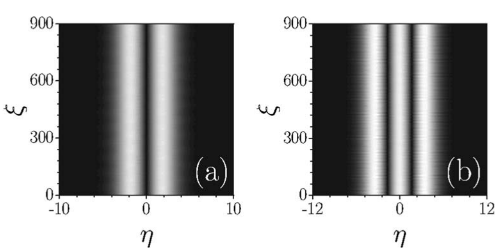

Fig. 4. Stable propagation of (a) dipole-mode and (b) triple-mode solitons that correspond to $b=1.5$ and $d=5$ in the presence of white input noise with variance $\sigma_{\text{noise}}^2=0.01$.

Summarizing, we investigated the stability of multiple-mode solitons in focusing, nonlocal Kerr-type nonlinear media. We found that, in a medium with an exponential nonlocal response, bound states are stable if they contain fewer than five solitons. Results motivate further study, e.g., about the generation and mobility of soliton trains.[21,22]

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