Inhibition of Light Tunneling in Waveguide Arrays

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We report the observation of almost perfect light tunneling inhibition at the edge and inside laser-written waveguide arrays due to band collapse. When the refractive index of the guiding channels is harmonically modulated along the propagation direction and out-of-phase in adjacent guides, light is trapped in the excited waveguide over a long distance due to resonances. The phenomenon can be used for tuning the localization threshold power.

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The precise control of wave packet tunneling by external driving fields is of major relevance in many branches of physics, such as superconducting quantum interference devices, spin systems, multiquantum dots, and cold atoms in optical traps [1,2]. Two different phenomena attract particular interest: dynamic localization in longitudinally periodic potentials [3–5] and driven double-well potentials [6,7] which are well suited to investigating tunneling control. Optical settings provide a new system to explore tunneling phenomena [8,9] as well as diffraction-free wave packet propagation [10,11]. In this regard, a particularly important system is put forward by arrays of evanescently coupled waveguides, where it was shown that the concept of inhibited light tunneling is possible not only via lattice soliton formation [12–15] but also due to a harmonic bending of the waveguides yielding either dynamic localization [16–20] or coherent destruction of tunneling [21,22]. However, while dynamic localization occurs only in systems without boundary interaction, coherent destruction of tunneling was achieved only in a two-waveguide system due to analogies with surface states in curved lattices [23].

In this Letter we demonstrate that a harmonic out-of-phase modulation of the linear refractive index along the propagation direction yields the almost perfect inhibition of the light tunneling between adjacent guiding channels irrespective of the input position in finite and infinite arrays. When the frequency and amplitude of the modulation are properly chosen, the band of quasienergies is considerably narrowed, forcing the light to remain in the excited channel. This phenomenon is possible in the coupler geometry, at the edge and in the interior of waveguide arrays. At intermediate power levels, the light partially delocalizes and eventually relocates again due to soliton formation at high power levels.

To gain intuitive insight, we start our analysis by studying the dimensionless equations describing propagation of light in the waveguide array in tight-binding approximation:

\[
i \frac{dq_m}{d\xi} + (-1)^m \mu \sin(\Omega \xi)q_m + C(q_{m+1} + q_{m-1}) + \chi q_m|q_m|^2 = 0.
\]

which describes the evolution of the amplitude in the \(n\)th waveguide \(q_m\), with light tunneling into adjacent guides with the tunneling rate \(C\) and the nonlinearity constant \(\chi\). The value \(1 > \mu > 0\) is the relative depth of the harmonic longitudinal modulation, while \(\Omega\) is its spatial frequency. The modulation of the refractive index between the adjacent guiding channels is out of phase. The transformation \(h_m = q_m \exp(i(-1)^m \mu \cos(\Omega \xi)/\Omega)\) yields

\[
i \frac{dh_m}{d\xi} + C(h_{m+1} + h_{m-1}) \exp(2i\mu \cos(\Omega \xi)/\Omega) + \chi h_m|h_m|^2 = 0.
\]

When using the expansion \(\exp(2i\mu \cos(\Omega \xi)/\Omega) = \sum_{k=0}^\infty i^k J_k(2\mu/\Omega) \exp(ik\Omega \xi)\) in terms of Bessel functions and neglecting all orders except \(k = 0\), one finds that diffraction vanishes when \((2\mu/\Omega) = v_j\) with \(v_j = 2.4, 5.5, \ldots\) being roots of the zero-order Bessel function. Hence, for a fixed modulation depth \(\mu\), such crude approximation predicts that there are resonance frequencies at which light tunneling is inhibited. In the limiting case of a linear optical coupler (\(\chi \rightarrow 0\)) when only the first channel is exited with unit amplitude, the solution of Eq. (2) is \(|h_1(\xi)|^2 = [1 + \cos(2\kappa \xi)]^2/2\), where the coupling constant \(\kappa = CJ_0(2\mu/\Omega)\) is reduced by the factor \(J_0(2\mu/\Omega)\).

The distance-averaged power fraction guided in the excited channel \(U_m = L^{-1} \int_0^L h_m(\xi)^2 d\xi\) can be found analytically as \(U_m = \{1 + \sin[2CLJ_0(2\mu/\Omega)]/2\}.\) Thus, around the zeros of the Bessel function, the power fraction can be estimated as \(U_m \approx 1 - C^2L^2J_0^2(2\mu/\Omega)/3\). When \(J_0(2\mu/\Omega) \rightarrow 0\), the localization is complete. In the non-
line,
When these results were analyzed for a waveguide array ($M = 13$, as in the experiment), we found out that the linear resonance curves are qualitatively similar for excitations of the edge channel and central channel [compare curves 1 and 2 in Fig. 2(d)]. However, the principal peak is more pronounced in the case of the edge channel excitation, because of the diminished discrete diffraction. Besides the most pronounced principal resonance at $\Omega_r \approx 1.30\Omega_b$, additional weaker peaks appear close to the fractional frequencies $\Omega_r/2, \Omega_r/3, \ldots$. The data of simulations were used for the optimization of the modulation frequency of fabricated arrays. We found numerically that for a modulation depth of $\mu = 0.2$ the optimal longitudinal frequency is $\Omega_r \approx 1.30\Omega_b$ for the surface channel excitation and $\Omega_r \approx 1.38\Omega_b$ when a waveguide in the array center is excited. Figure 3 compares the light propagation in nonmodulated and optimally modulated waveguide arrays for the edge [panels (a), (b)] and central channel [panels (c), (d)] excitations. This is a generalization of the tunneling control in a double-well potential. Because of the modulation of the refractive index the Floquet-Bloch modes exhibit almost identical quasienergies, irrespective of the number of waveguides in the system or the position of excitation. Note that the possibility of linear light localization in the bulk or at the surface of arrays expands the opportunities for diffraction control and spatiotemporal selectivity of light localization.

To observe the impact of nonlinearity we monitored the power-dependent tunneling inhibition with femtosecond-pulsed radiation ($\tau_{\text{pulse}} = 150$ fs) of a Ti:sapphire laser. In

![Figure 2](image1.png)

**FIG. 2.** (a) $\Omega_r/\Omega_b$ versus $\mu$ for a linear coupler. (b) $U_m$ versus $\Omega_r/\Omega_b$ in a linear coupler at $\mu = 0.2$. (c) $U_m$ versus $A^2$ in a coupler at $\mu = 0.1$ and $\Omega_r/\Omega_b = 0.76$. (d) $U_m$ in the surface channel (curve 1) or in the bulk (curve 2) of a linear array versus $\Omega_r/\Omega_b$ at $\mu = 0.2$.

As mentioned above, nonlinearity slows down the power oscillations (see [26] for details). Therefore, if the modulation frequency is equal to or lower than the resonant one ($\Omega \leq \Omega_r$), an increase of the normalized peak intensity $A^2$ of the input beam shifts the power oscillations’ frequency away from the resonance peak, thus leading to initial delocalization, while relocalization appears at higher powers due to soliton formation [see Fig. 2(c) where $\Omega$ matches the principal linear resonance]. For $\mu = 0.1$ when $\Omega_r = 0.76\Omega_b$, the minimal localization corresponds to different amplitudes $A^2 = 0.70, 0.82, 0.90$ for $\Omega_r/\Omega_b = 0.66, 0.71, 0.76$, respectively, but for all these frequencies relocalization occurs approximately for the same amplitude $A^2 \approx 1.05$. In contrast, when $\Omega > \Omega_r$, nonlinearity shifts the frequency of power oscillations towards resonant values, thus producing localization enhancement from the very beginning. For example, at $\mu = 0.1, \Omega_r = 0.76\Omega_b$ the localization maximum appears at $A^2 \approx 0.46$ and $0.70$ for $\Omega_r/\Omega_b = 0.81$ and $0.86$, respectively. These amplitudes are smaller than the critical value $A^2 \approx 1.4$ at which localization occurs in the unmodulated coupler ($\mu = 0$), a result that indicates that out-of-phase longitudinal modulation of the refractive index might be used for fine-tuning the localization threshold power.
Um responds to the simulations of the power dependence of the soliton formation at high input power. This behavior correlates to delocalization at intermediate power level, and finally nonmodulated array to a soliton-type tunneling inhibition (a) and (c) show the transformation of light tunneling in a waveguide array. Columns of the experimentally observed output patterns. Columns (a) and (c) show the transformation of light tunneling in a nonmodulated array to a soliton-type tunneling inhibition with increasing input power; columns (b) and (d) illustrate linear tunneling inhibition in a modulated array, partial delocalization at intermediate power level, and finally soliton formation at high input power. This behavior corresponds to the simulations of the power dependence of the localization parameter $U_w$. At low power, the resonant light propagation results in the inhibition of light diffraction. For an increased intermediate power, the nonlinear influence distorts tunneling inhibition, so that light can couple from the excited into the adjacent guides. However, at high input power, a solitonlike localization occurs due to the Kerr effect. It is interesting to note that this is a representation of a diffraction-managed soliton, which was demonstrated experimentally only recently [27]. However, in our system it is possible to obtain soliton formation not only for higher power (when the resonance condition is satisfied) but also for decreased power (when the propagation is slightly off-resonant and the resonance curve is broadened by the nonlinear influence).

In conclusion, we observed experimentally light tunneling inhibition in waveguide arrays with a harmonic out-of-phase longitudinal modulation of the refractive index. The setup is a generalization of a double-well potential and allows full control of tunneling in an extended potential. The results indicate that resonant phenomena accessible in longitudinally modulated structures open new ways for the control of light propagation.

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