Disorder-induced phase control in superfluid Fermi-Bose mixtures

2009 Europhys. Lett. 86 26004

(http://iopscience.iop.org/0295-5075/86/2/26004)

A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

April 2009

EPL, **86** (2009) 26004
doi: 10.1209/0295-5075/86/26004

www.epljournal.org

Disorder-induced phase control in superfluid Fermi-Bose mixtures

A. NIEDERBERGER[1], J. WEHR[1,2], M. LEWENSTEIN[1,3] and K. SACHA[1,4]

[1] *ICFO-Institut de Ciències Fotòniques, Mediterranean Technology Park - 08860 Castelldefels (Barcelona), Spain, EU*
[2] *Department of Mathematics, University of Arizona - Tucson, AZ 85721-0089, USA*
[3] *ICREA-Instituciò Catalana de Ricerca i Estudis Avançats - 08010 Barcelona, Spain, EU*
[4] *Marian Smoluchowski Institute of Physics and Mark Kac Complex Systems Research Centre, Jagiellonian University Reymonta 4, 30059 Kraków, Poland, EU*

received 29 August 2008; accepted in final form 20 March 2009
published online 5 May 2009

PACS 67.85.Pq – Mixtures of Bose and Fermi gases
PACS 64.60.Cn – Order-disorder transformations
PACS 03.75.Ss – Degenerate Fermi gases

Abstract – We consider a mixture of a superfluid Fermi gas of ultracold atoms and a Bose-Einstein condensate of molecules possessing a continuous $U(1)$ (relative phase) symmetry. We study the effects of a spatially random photo-associative–dissociative symmetry-breaking coupling of the systems. Such coupling allows one to control the relative phase between a superfluid order parameter of the Fermi system and the condensate wave function of molecules for temperatures below the Bardeen-Cooper-Schriefer critical temperature. The presented mechanism of phase control belongs to the general class of phenomena in which disorder interacts with continuous symmetry. Our results show the robustness and wide range of applicability of disorder-induced order and are valid for both disordered and regular couplings. Here, the effect is studied in the case of interacting fermionic and bosonic gases in the superfluid phase.

Introduction. – Experiments on ultracold atoms are allowing for realizations of highly complex mathematical models used in a variety of fields in physics. For example, condensed matter models like the XY model can be realized using bosons on an optical lattice [1] and coupled nonlinear differential equations can be found in laser-physics as well as in models describing Bose-Einstein condensates [2]. Of all the studies of the physics of ultracold atoms, fermionic systems and, in particular, Fermi superfluids have recently received a lot of attention. The most famous example in this context is the BEC-BCS crossover. In this example, fermions are transformed between the superconducting state in which they form Bardeen-Cooper-Schriefer (BCS) pairs and the Bose-Einstein condensate (BEC) made of diatomic molecules of the same fermions (for reviews see [3–5]). Another field of particularly high activity has recently been the study of disorder in different fields of physics (see, for example, [6]). Since the proposal to study disorder in degenerate quantum gases [7], several experimental and theoretical groups have focused on this subject [8–17]. Very recently, these studies culminated in the seminal experimental observation of Anderson localization of matter waves in a BEC with a disordered potential [18,19] and in signatures of the Bose glass state of an ultracold gas in an optical lattice [20].

Recently, we have proposed another kind of disorder effects that can be realized with ultracold atoms, termed *disorder-induced order control*. Generally speaking, these effects occur in systems possessing a continuous symmetry. For low-dimensional systems of this sort (such as the homogeneous XY model in 2D), the Mermin-Wagner-Hohenberg (MWH) theorem [21], prevents long-range order at non-zero temperatures. The system does order at $T=0$, but arbitrarily small disorder with a distribution respecting the continuous symmetry destroys this $T=0$ order (see [22] and refs. therein). Adding a disorder with a distribution that breaks the continuous symmetry leads to competing mechanisms: Naturally, disorder acts against ordering, whereas symmetry breaking increases the tendency to order. We have shown rigorously that in the classical XY model in a random X-oriented magnetic field, a spontaneous magnetization in the Y-direction appears at $T=0$. Also, we have presented strong evidence for the persistence of magnetization at $T>0$ in the limit of small disorder. Clearly, in this case we are dealing with *disorder-induced order*.

The situation is somewhat different when we deal with systems that order at $T_c > T > 0$ (such as the homogeneous XY or Heisenberg model in 3D, or the inhomogeneous XY model in 2D, corresponding to the trapped 2D BEC). Adding a small disorder with a distribution respecting the symmetry will generally not do much in this case, except reduce T_c. A rigorous theorem (valid also at $T = 0$, [22]) says that the order parameter conjugate to an arbitrarily small added random field has a vanishing quenched average over the disorder. Thus, if the disorder is, say, X oriented, we obtain that the averaged X-component of the magnetization vanishes. This may happen because it fluctuates a lot, with the fluctuations averaging to zero, or because it is typically close to zero. In the latter case, *disorder prevents order in the X-direction*; the systems tends, however, to order in a perpendicular direction, and that is why we termed this situation *disorder-induced order control*.

A related but distinct effect occurs in a two-component BEC with a random Raman coupling. In this case, turning on a small real-valued random coupling between the two components causes their order parameters to be perpendicular, *i.e.*, the phase difference between the two condensate wave functions is $\pi/2$. while this can be seen as an order phenomenon, we stress that from the point of view of the general theorem in [22], it relies on *vanishing* of the quantity conjugate to the random couplings, namely the inner product of the two order parameters, a mechanism different from the one studied in [1].

Similar effects can be found in other areas of physics as described, for example in [23]. These effects occur for all types of symmetry-breaking perturbations with mean zero: random couplings, but also regularly oscillating, or pseudo-random couplings. The term disorder-induced order was chosen to stress the presence of the effect in the most non-trivial case.

The main goal of the present letter is to introduce these new ideas to the field of superfluid Fermi gases that possess a $U(1)$ phase symmetry. Even if we use direct analogy between this system and the case of two BECs or (a less direct) analogy to the XY model in a uniaxial field, as discussed in refs. [1,24], it is important to understand that the underlying physical systems and mathematical models, as well as the numerical and analytical methods of these different cases, are very different. Note that the disorder effects are hardly visible for the homogeneous XY models, but very spectacular in the case of the coupled BECs. It is by no means *a priori* obvious how pronounced they will be in the Fermi case.

Here, we show that a Fermi superfluid coupled to a molecular BEC *via* a random, symmetry-breaking photoassociating-dissociating coupling undergoes relative phase ordering, so that the phase of the order parameter can be efficiently controlled by the phase of the coupling. Within a BCS-like theory [25], we show rigorously that for small disorder the effect is large and robust, and that it occurs practically for all temperatures below the superfluid transition temperature.

Model description. – We consider a mixture of fermions in two different internal states interacting *via* an attractive zero-range potential in a 3D volume V with a Hamiltonian

$$H_{\mathrm{F}} = \int \mathrm{d}\mathbf{r} \left[\sum_j \hat{\psi}_j^\dagger \left(-\frac{\hbar^2}{2m}\nabla^2 - \mu \right) \hat{\psi}_j - g\hat{\psi}_1^\dagger\hat{\psi}_2^\dagger\hat{\psi}_2\hat{\psi}_1 \right], \tag{1}$$

where the chemical potential μ fixes the average density n, $g = 4\pi\hbar^2|a|/m$ (with s-wave scattering length a) determines the strength of the interactions and $\hat{\psi}_j$ stand for the fermionic field operators. Moreover, we assume the presence of a BEC of molecular dimers consisting of the two fermionic species and a (weak) coupling that transforms these dimers into fermion pairs and *vice versa*. Experimentally, we can sweep a mixture of fermions with two different internal states over a Feshbach resonance, which leaves us with a BEC of diatomic molecules and unbound fermions. Then, we approach a second Feshbach resonance which turns the previously unbound fermions into BCS pairs without affecting the molecular BEC. For example, potassium-40 has known s-wave resonances at 202G and 224G with widths of the order of 10G [26] and might be a realistic candidate for such experiments. The coupling between the molecules and the fermions can be realized through photoassociation and photodissociation. Taking the limit of a large BEC, we do not need to consider its dynamics because the effect of the weak coupling with the fermions on BEC is negligible. Following these considerations, the Hamiltonian (1) has to be supplemented by one term only: The coupling between the fermions and the BEC which we approximate by $\int \mathrm{d}\mathbf{r}[\alpha^*(\mathbf{r})\hat{\psi}_d^\dagger\hat{\psi}_1\hat{\psi}_2 + \alpha(\mathbf{r})\hat{\psi}_2^\dagger\hat{\psi}_1^\dagger\hat{\psi}_d] \approx \sqrt{n_d}\int \mathrm{d}\mathbf{r}[\alpha^*(\mathbf{r})\hat{\psi}_1\hat{\psi}_2 + \alpha(\mathbf{r})\hat{\psi}_2^\dagger\hat{\psi}_1^\dagger]$, where the bosonic field operator $\hat{\psi}_d$ for molecules is substituted by a real-valued condensate wave function which for a homogeneous case considered here is the square root of the density of dimers $\sqrt{n_d}$. The full Hamiltonian therefore reads

$$H = H_{\mathrm{F}} + \int \mathrm{d}\mathbf{r} \left[\Gamma^*(\mathbf{r})\hat{\psi}_1\hat{\psi}_2 + \Gamma(\mathbf{r})\hat{\psi}_2^\dagger\hat{\psi}_1^\dagger \right], \tag{2}$$

with $\Gamma(\mathbf{r}) = \tilde{\Gamma}(\mathbf{r})\mathrm{e}^{-\mathrm{i}\varphi_\Gamma} = \sqrt{n_d}\alpha(\mathbf{r})$. We assume that the transfer process is realized so that $\tilde{\Gamma}(\mathbf{r})$ is real, varies randomly in space and is constant in time, $\int \mathrm{d}\mathbf{r}\tilde{\Gamma}(\mathbf{r}) = 0$ and φ_Γ is a real number. We will show that for $\varphi_\Gamma = 0$ the relative phase between the condensate wave function of molecules and the pairing function of the superfluid fermions is fixed to $\pi/2$ (or $-\pi/2$). Then we show that we can control the relative phase and fix it to any value by changing a control parameter φ_Γ, and the relative phase will be different from φ_Γ by $\pm\pi/2$. This is in stark contrast to the case of constant coupling of strength c, *i.e.*, $\Gamma(\mathbf{r}) \equiv c$, where the relative phase trivially follows the phase of

$\Gamma(\mathbf{r})$. For $\Gamma=0$ and in the weak-coupling limit, *i.e.*, $g\to 0$, we deal with a Fermi system which for T below the critical temperature $T_{\rm c}=8\,{\rm e}^{\gamma}{\rm e}^{-2}\pi^{-1}T_{\rm F}{\rm e}^{-\pi/2k_{\rm F}|a|}$ (where $k_{\rm B}T_{\rm F}=\varepsilon_{\rm F}=\hbar^2k_{\rm F}^2/2m=\hbar^2(3\pi^2n)^{2/3}/2m$ and $\gamma=0.5772$ the Euler constant) reveals a transition to a superfluid phase (BCS state) which is indicated by a non-vanishing pairing function (order parameter) $\Delta=g\langle\hat\psi_2\hat\psi_1\rangle$ [25,27]. In the general case (*i.e.*, including non-zero Γ), the pairing function is given in terms of solutions of Bogoliubov-de Gennes (BdG) equations, *i.e.*, $\Delta=g\sum_n u_nv_n^*[1-2f(E_n)]$, where

$$\begin{bmatrix}-\frac{\hbar^2\nabla^2}{2m}-\mu+W & \Delta+\Gamma\\ \Delta^*+\Gamma^* & \frac{\hbar^2\nabla^2}{2m}+\mu-W\end{bmatrix}\begin{bmatrix}u_n\\ v_n\end{bmatrix}=E_n\begin{bmatrix}u_n\\ v_n\end{bmatrix}\qquad(3)$$

with a Hartree-Fock term $W=-g\langle\hat\psi_1^\dagger\hat\psi_1\rangle=-g\langle\hat\psi_2^\dagger\hat\psi_2\rangle=-g\sum_n\left(|u_n|^2f(E_n)+|v_n|^2[1-f(E_n)]\right)$ and the Fermi distribution $f(E_n)=({\rm e}^{E_n/k_{\rm B}T}+1)^{-1}$ [25].

If the transfer process is absent (*i.e.*, $\Gamma=0$), the system (2) is invariant under global gauge transformation, *i.e.*, $\hat\psi_j\to{\rm e}^{{\rm i}\varphi/2}\hat\psi_j$, which implies that if $\{u_n,v_n\}$ are solutions of the BdG equations for Δ, then $\{{\rm e}^{{\rm i}\varphi/2}u_n,\,{\rm e}^{-{\rm i}\varphi/2}v_n\}$ are the solutions corresponding to ${\rm e}^{{\rm i}\varphi}\Delta$. This continuous symmetry is broken when the transfer process is turned on, as can be seen from (2). Then the phase of the pairing function becomes relevant because it is the relative phase with respect to the (real-valued) condensate wave function of dimers.

Theoretical study. – We begin with an analysis of the $\varphi_\Gamma=0$ case, *i.e.*, for $\Gamma=\tilde\Gamma$ real. Let us assume that for $\Gamma=0$ and for some temperature T, we have a non-zero pairing function that is chosen to be real and positive, $\Delta_0>0$. When we turn on Γ with $|\Gamma(\mathbf{r})|\ll\Delta_0$, we may expect that it results in a new pairing function where $\Delta(\mathbf{r})\approx\Delta_0{\rm e}^{{\rm i}\varphi(\mathbf{r})}$. That is, any non-zero Γ has a dramatic effect on the phase because without the transfer process the system is degenerate with respect to the choice of φ. On the other hand, an infinitesimal Γ is not able to change $|\Delta(\mathbf{r})|$ because this would cost energy. Moreover, we may expect that $\varphi(\mathbf{r})$ oscillates around some average value φ_0 with small amplitude because we assume that $\Gamma(\mathbf{r})$ fluctuates around zero with infinitesimal variance. Under these assumptions, we can observe that $|\varphi_0|=\pi/2$. In fact, let us neglect the Hartree-Fock term W (which is not essential for Fermi superfluidity) and calculate the difference of the thermodynamic potentials between the superfluid and the normal phase

$$\begin{aligned}\Omega_{\rm s}-\Omega_0 &= \int_0^1\frac{{\rm d}\lambda}{\lambda}\langle\lambda H_1\rangle_\lambda\\ &\approx -\int_0^g\frac{{\rm d}g'}{g'^2}\int{\rm d}\mathbf{r}\left[|\Delta|^2+\frac{2g'}{g}\tilde\Gamma|\Delta|\cos\varphi\right],\end{aligned}\qquad(4)$$

where $H_1=\int{\rm d}\mathbf{r}\left[\tilde\Gamma(\hat\psi_1\hat\psi_2+\hat\psi_2^\dagger\hat\psi_1^\dagger)-g\hat\psi_1^\dagger\hat\psi_2^\dagger\hat\psi_2\hat\psi_1\right]$ [26]. According to our assumptions, for g' close to g, $|\Delta(\mathbf{r})|$ is constant and $\cos\varphi(\mathbf{r})\approx\cos\varphi_0-\sin\varphi_0\delta\varphi(\mathbf{r})$. Then

$$\begin{aligned}-\int{\rm d}\mathbf{r}\left[|\Delta|^2+\frac{2g'}{g}\tilde\Gamma|\Delta|\cos\varphi\right]\approx\\ -|\Delta|^2V+\sin\varphi_0\frac{2g'|\Delta|}{g}\int{\rm d}\mathbf{r}\tilde\Gamma\delta\varphi,\end{aligned}\qquad(5)$$

and for $\int{\rm d}\mathbf{r}\tilde\Gamma\delta\varphi<0$ the thermodynamic potential is minimized when $\varphi_0=\pi/2$. With the transformation $\delta\varphi\to-\delta\varphi$ and $\varphi_0\to-\pi/2$, we obtain another solution which reflects the symmetry of the system. That is, for a real Γ, if $\Delta(\mathbf{r})$ corresponds to solution of (3) then the solution of complex conjugate BdG equations results in a new pairing function equal to $\Delta^*(\mathbf{r})$. In experiments, the sign of φ_0 will depend on the realization and is determined by spontaneous breaking of the $\varphi\to-\varphi$ symmetry.

It is important to note that eq. (5) shows that disorder-induced order is present for very different types of couplings: For regularly oscillating, pseudo-random, and random couplings, the effect is present as long as the mean value of $\tilde\Gamma$ is zero and the resulting fluctuations are small, *i.e.*, $|\delta\varphi(\mathbf{r})|\ll\pi$.

Having determined φ_0 we would like to estimate fluctuations of the phase of the pairing function $\delta\varphi(\mathbf{r})$. To this end, let us employ the Ginzburg-Landau (GL) approach [25]. Adapting the Gorkov's derivation of the GL equation [25,28,29] (with the standard regularization of the bare interaction g for the case of cold atomic gases) to our problem we obtain

$$\begin{aligned}\nabla^2\Delta &= -\nabla^2\tilde\Gamma-\frac{48\pi^2}{7\zeta(3)l_c^2}\left(\frac{2\pi^2\hbar^2}{mk_{\rm F}g}+\frac{T_{\rm c}-T}{T_{\rm c}}\right)\tilde\Gamma\\ &\quad-\frac{48\pi^2}{7\zeta(3)l_c^2}\frac{T_{\rm c}-T}{T_{\rm c}}\Delta+\frac{6m^2}{\hbar^4k_{\rm F}^2}|\Delta+\tilde\Gamma|^2(\Delta+\tilde\Gamma),\end{aligned}\qquad(6)$$

where $l_c=\hbar^2k_{\rm F}/mk_{\rm B}T_c$. Equation (6) is valid for $T_{\rm c}-T\ll T_{\rm c}$ and for $\tilde\Gamma(\mathbf{r})$ that changes on a scale much larger than l_c (*e.g.*, for $k_{\rm F}|a|=0.5$ and $n\sim10^{14}\,{\rm cm}^{-3}$ we get $l_c\sim4\,\mu{\rm m}$). For $|\tilde\Gamma(\mathbf{r})|$ much smaller than $|\Delta_0(T)|$, where $\Delta_0(T)$ is the pairing function in the absence of the transfer process, we may introduce further approximations that reduce eq. (6) to

$$|\Delta_0|\nabla^2\delta\varphi(\mathbf{r}) = \nabla^2\tilde\Gamma(\mathbf{r})+\frac{48\pi^2}{7\zeta(3)l_c^2}\left(\frac{2\pi^2\hbar^2}{mk_{\rm F}g}+\frac{T_{\rm c}-T}{T_{\rm c}}\right)\tilde\Gamma(\mathbf{r}),\qquad(7)$$

where $|\Delta|\approx|\Delta_0|$ and we have chosen $\varphi_0=\pi/2$ in the expansion $\varphi(\boldsymbol{r})\approx\varphi_0+\delta\varphi(\mathbf{r})$. The solution of (7) reads

$$\delta\varphi(\mathbf{k}) = \frac{\tilde\Gamma(\mathbf{k})}{|\Delta_0|}-\frac{48\pi^2}{7\zeta(3)l_c^2|\Delta_0|}\left(\frac{2\pi^2\hbar^2}{mk_{\rm F}g}+\frac{T_{\rm c}-T}{T_{\rm c}}\right)\frac{\tilde\Gamma(\boldsymbol{k})}{|\mathbf{k}|^2}\qquad(8)$$

in the Fourier space.

Now we switch to the general case of complex $\Gamma=\tilde\Gamma{\rm e}^{-{\rm i}\varphi_\Gamma}$. It is easy to check that if $|\Delta|{\rm e}^{{\rm i}\varphi}$ corresponds to solution of the BdG equations with $\varphi_\Gamma=0$ then

$|\Delta|e^{i(\varphi-\varphi_\Gamma)}$ is related to the solution for $\varphi_\Gamma \neq 0$. This implies that, if for $\varphi_\Gamma = 0$, we are able to fix the relative phase between the condensate wave function of molecules and the pairing function of the superfluid fermions to $\pi/2$ (or $-\pi/2$), then changing φ_Γ allows us to fix it to $\phi_0 = \pi/2 - \varphi_\Gamma$ (or $\phi_0 = -\pi/2 - \varphi_\Gamma$) and phase control emerges.

Numerical study. – Assuming that the transfer process with small $|\Gamma(\mathbf{r})|$ results in phase fluctuations of $\Delta(\mathbf{r})$ only, we have shown that the fluctuations occur around $\pi/2 - \varphi_\Gamma$ (or $-\pi/2 - \varphi_\Gamma$) and they are given by eq. (8). Now we would like to switch to numerical solutions of the BdG equations (where, in contrast to the analytical study, we do not neglect the Hartree-Fock term W) to demonstrate that indeed for $|\Gamma| \ll |\Delta_0|$ the fluctuations are small and the predicted phase control is possible. In 3D calculations we regularize the coupling constant g in $\Delta = g\langle\hat{\psi}_2\hat{\psi}_1\rangle$, *i.e.*, $g \to g_{\text{eff}}$, according to

$$\frac{1}{g_{\text{eff}}} = \frac{1}{g} - \frac{mk_{\text{F}}}{2\pi^2\hbar^2}\left(\frac{1}{2}\ln\frac{\sqrt{E_{\text{C}}}+\sqrt{\varepsilon_{\text{F}}}}{\sqrt{E_{\text{C}}}-\sqrt{\varepsilon_{\text{F}}}} - \sqrt{\frac{E_{\text{C}}}{\varepsilon_{\text{F}}}}\right), \qquad (9)$$

where the logarithmic term results from the sum over Bogoliubov modes corresponding to energy above the cut-off E_{C} performed in the spirit of the local density approximation, see [30] for details. For the simulations we choose $L_z = 40k_{\text{F}}^{-1}$, $L_\perp = 20k_{\text{F}}^{-1}$, $\mu = 0.83\varepsilon_{\text{F}}$, and $k_{\text{F}}|a| = 0.4$, which for $\Gamma = 0$ and the cut-off $E_{\text{C}} = 100\varepsilon_{\text{F}}$ leads to $\Delta_0(T=0) = 0.036\varepsilon_{\text{F}}$ and $T_{\text{c}} = 0.019T_{\text{F}}$. Using these parameters $l_c \sim 100k_{\text{F}}^{-1}$ is larger than the system size and we are able to explore a regime beyond GL theory. We assume real $\Gamma(\mathbf{r})$ given by a pseudo-random function that changes along the z-axis only,

$$\Gamma(\mathbf{r}) = \frac{\Gamma_0}{2}\left[\sin\left(\frac{2\pi}{L_z}(9z+8.8)\right) + \sin\left(\frac{2\pi}{L_z}(13z+3.6)\right)\right]. \qquad (10)$$

In fig. 1, we show the phase of the pairing function $\varphi(z)$ in the case when $\Gamma_0 = 0.01|\Delta_0(0)|$ and $\varphi_\Gamma = 0$ for two different temperatures, $T = 0$ and $T = 0.9T_{\text{c}}$. One can see that indeed the phase oscillates around $\pi/2$ with a small amplitude (standard deviation of the order 10^{-2}). The fluctuations of the absolute value of $\Delta(z)$ are negligible (standard deviations divided by average values are of the order 10^{-4}). When T approaches T_{c} the average $|\Delta|$ decreases and, at some T, becomes much smaller than Γ_0 and we enter another regime where the transfer term in the Hamiltonian (2) starts dominating. For a very large Γ_0 we may expect that a real-valued Δ, which oscillates in space with a phase approximately opposite to the one of $\Gamma(z)$, minimizes the thermodynamic potential. In fig. 2, we present average values and standard deviations for φ and $|\Delta|$ *vs.* temperature where one can observe an increase of the fluctuations for $T \to T_{\text{c}}$, which is typical for critical phenomena [31].

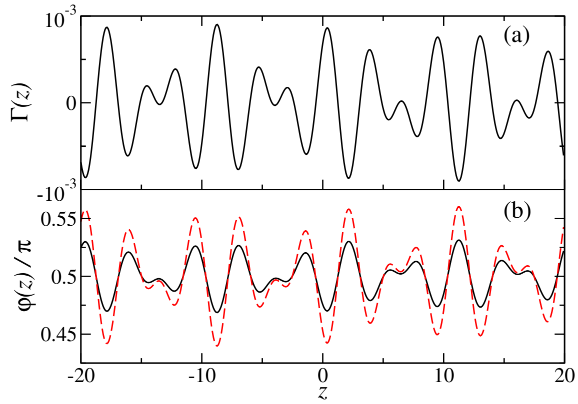

Fig. 1: (Color online) Panel (a) shows $\Gamma(z)$ given in eq. (10) for $\Gamma_0 = 0.01|\Delta_0(0)|$. Panel (b) represents the corresponding phase $\varphi(z)$ of the pairing function for $T = 0$ (black solid curve) and $T = 0.9T_{\text{c}}$ (red dashed curve).

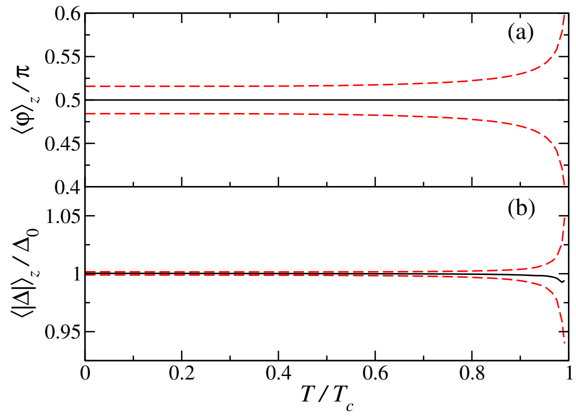

Fig. 2: (Color online) Panel (a) shows average value of the phase of the pairing function $\langle\varphi\rangle_z$ *vs.* temperature obtained for Γ as in fig. 1. In panel (b) we present the corresponding average value of the modulus of the pairing function $\langle|\Delta|\rangle_z$ divided by $\Delta_0(T)$, *i.e.*, the pairing for the $\Gamma = 0$ case. Solid black curves are related to average values, dashed red curves to average values $\pm$ standard deviation. The figure shows simulations for temperatures up to $T = 0.99T_{\text{c}}$, where $\Delta_0(T) = 0.16\Delta_0(0)$.

Summary. – We have shown how to control the relative phase φ between the wave function of a molecular condensate and the pairing function of a mixture of fermions in the BCS state. It turns out that weak couplings of a certain class which transfer pairs of fermions into molecules and *vice versa*, fix this relative phase. Contrary to phase control using constant couplings, disorder-induced phase control employs spatially randomly varying or oscillating couplings; they can be realized by optical means, with a desired phase and amplitude, which allows for efficient control of φ. In this letter, we have considered the Fermi system in a weak-coupling regime but similar behavior is expected in the strong regime. In particular, translation of our results to the simplified resonant superfluidity theory (cf. [32]) is straightforward.

Our results hold also for $0 < k_{\mathrm{F}} a \ll 1$, where the pairing function becomes a condensate wave function of tightly bound pairs. Hence, the present situation turns out to be similar to the control of the relative phase between two Bose-Einstein condensates, analyzed in our earlier publication [24]. The problem considered here also belongs to a general class of disorder-induced order phenomena that rely on continuous symmetry breaking.

* * *

We acknowledge support of the EU IP Programme "Scala", the ESF Programmes "Qudedis", "Fermix", the Spanish MEC grants (FIS 2005-04627, FIS 2008-00784 Consolider Ingenio 2010 "QOIT") and the Alexander von Humboldt Foundation. JW was partially supported by the NSF grant DMS 0623941; he gratefully acknowledges the hospitality of L. Torner and ICFO. KS acknowledges Polish Government scientific funds (2008–2011) as a research project and the Marie Curie ToK project COCOS (MTKD-CT-2004-517186). The research was partially conducted within the LFPPI network.

REFERENCES

[1] Wehr J., Niederberger A., Sanchez-Palencia L. and Lewenstein M., *Phys. Rev. B*, **74** (2006) 224448.
[2] Staliunas K. *et al.*, *Opt. Commun.*, **268** (2006) 160.
[3] Inguscio M., Ketterle W. and Salomon C. (Editors), *Proceedings of the International School of Physics Enrico Fermi, Varenna, 2006, Course CLXIV* (IOS Press, Amsterdam) 2007.
[4] Bloch I., Dalibard J. and Zwerger W., *Rev. Mod. Phys.*, **80** (2008) 885.
[5] Giorgini S., Pitaevskii L. P. and Stringari S., *Rev. Mod. Phys.*, **80** (2008) 1215.
[6] Kuhn R. C., Miniatura C. and Delande D. *et al.*, *Phys. Rev. Lett.*, **95** (2005) 250403; Nemec N., Richter K. and Cuniberti G., *New J. Phys.*, **10** (2008) 065014; Schwartz T., Bartal G., Fishman S. *et al.*, *Nature*, **446** (2007) 7131; Fischer B., Rosen A. and Fishman S., *Opt. Lett.*, **24** (1999) 21; Fiebig S., Aegerter C. M., Buhrer W. *et al.*, *EPL*, **81** (2008) 64004.
[7] Damski B. *et al.*, *Phys. Rev. Lett.*, **91** (2003) 080403; Roth R. and Burnett K., *Phys. Rev. A*, **68** (2003) 023604.
[8] Lewenstein M. *et al.*, *Adv. Phys.*, **56** (2007) 243.
[9] Fallani L., Fort C. and Inguscio M., *Adv. At. Mol. Opt. Phys.*, **56** (2008) 119.
[10] Lye J. E. *et al.*, *Phys. Rev. Lett.*, **95** (2005) 070401; Fort C. *et al.*, *Phys. Rev. Lett.*, **95** (2005) 170410.
[11] Clément D. *et al.*, *Phys. Rev. Lett.*, **95** (2005) 170409; Clément D. *et al.*, *New J. Phys.*, **8** (2006) 165.
[12] Schulte T. *et al.*, *Phys. Rev. Lett.*, **95** (2005) 170411; Schulte T. *et al.*, *New J. Phys.*, **8** (2006) 230.
[13] Ahufinger V. *et al.*, *Phys. Rev. A*, **72** (2005) 063616.
[14] Sanchez-Palencia L., *Phys. Rev. A*, **74** (2006) 053625; Lugan P. *et al.*, *Phys. Rev. Lett.*, **98** (2007) 170403.
[15] Sanchez-Palencia L. *et al.*, *Phys. Rev. Lett.*, **98** (2007) 210401; Paul T. *et al.*, *Phys. Rev. Lett.*, **98** (2007) 210602.
[16] Bilas N. and Pavloff N., *Eur. Phys. J. D*, **40** (2006) 387.
[17] De Martino A. *et al.*, *Phys. Rev. Lett.*, **94** (2005) 060402; Roscilde T. and Cirac J. I., *Phys. Rev. Lett.*, **98** (2007) 190402.
[18] Billy J. *et al.*, *Nature*, **453** (2008) 891.
[19] Roati G. *et al.*, *Nature*, **453** (2008) 895.
[20] Fallani L. *et al.*, *Phys. Rev. Lett.*, **98** (2007) 130404.
[21] Mermin D. and Wagner H., *Phys. Rev. Lett.*, **17** (1966) 1133; Hohenberg P. C., *Phys. Rev.*, **158** (1967) 383.
[22] Aizenman M. and Wehr J., *Phys. Rev. Lett.*, **62** (1989) 2503; *Commun. Math. Phys.*, **130** (1990) 489.
[23] Aharony A., *Phys. Rev. B*, **18** (1978) 3328; Minchau B. J. and Pelcovits R. A., *Phys. Rev. B*, **32** (1985) 3081; Feldman D. E., *J. Phys. A*, **31** (1998) L177; Abanin D. A., Lee P. A. and Levitov L. S., *Phys. Rev. Lett.*, **98** (2007) 156801; Volovik G. E., *JETP Lett.*, **84** (2006) 455.
[24] Niederberger A., Schulte T., Wehr J., Lewenstein M., Sanchez-Palencia L. and Sacha K., *Phys. Rev. Lett.*, **100** (2008) 030403.
[25] Fetter A. L. and Walecka J. D., *Quantum Theory of Many-Particle Systems* (Dover Publications) 2003.
[26] Regal C. A. and Jin D. S., *Phys. Rev. Lett.*, **90** (2003) 23; Regal C. A., Greiner M. and Jin D. S., *Phys. Rev. Lett.*, **92** (2004) 4.
[27] Sá de Melo C. A. R., Randeria M. and Engelbrecht J. R., *Phys. Rev. Lett.*, **71** (1993) 3202.
[28] Gorkov L. P., *Sov. Phys. JETP*, **9** (1959) 1364.
[29] Baranov M. A. and Petrov D. S., *Phys. Rev. A*, **58** (1998) R801.
[30] Bruun G., Castin Y., Dum R. and Burnett K., *Eur. Phys. J. D*, **7** (1999) 433; Bulgac A. and Yu Y., *Phys. Rev. Lett.*, **88** (2002) 042504; Grasso M. and Urban M., *Phys. Rev. A*, **68** (2003) 033610.
[31] Herbut I., *A Modern Approach to Critical Phenomena* (Cambridge University Press) 2007.
[32] Holland M. *et al.*, *Phys. Rev. Lett.*, **87** (2001) 120406.