

Switching management in couplers with biharmonic longitudinal modulation of refractive index

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We address light propagation in couplers with longitudinal biharmonic modulation of refractive index in neighboring channels. While simplest single-frequency out-of-phase modulation allows suppression of coupling for a strictly defined set of resonant frequencies, the addition of modulation on multiple frequencies dramatically modifies the structure of resonances. Thus, additional modulation on a double frequency may suppress the primary resonance, while modulation on a triple frequency causes fusion of primary and secondary resonances. © 2009 Optical Society of America
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The optical materials with inhomogeneous refractive index landscapes provide exceptional opportunities for diffraction control, especially when the refractive index varies both in the transverse plane and in the direction of propagation of radiation [1,2]. In such structures discrete diffraction-managed solitons exist [3–5], while the propagation direction and transverse shape of the beam can be dynamically altered [6–9]. Among the most spectacular phenomena in longitudinally modulated structures [10–21] are resonant suppression of light coupling (analogous to coherent destruction of tunneling in double-well structures perturbed by a harmonic driving force [17]) and dynamic localization. Such phenomena are possible in waveguide arrays [10–16] and two-channel structures [17–21] when channels bend periodically in the propagation direction [10–12,18,19] or when the width or refractive index in neighboring channels oscillate out-of-phase [13–16,21]. To date only the simplest harmonic laws of longitudinal modulation were considered. Only recently stacking segments with different curvatures in periodically bending structures were used to observe polychromatic dynamic localization [22,23]. The suppression of coupling for narrow excitations in systems with multiple-frequency oscillations of refractive index was not addressed.

In this Letter we consider light propagation in directional couplers with complex out-of-phase “biharmonic” modulation of refractive index in neighboring channels. We show that the addition of even weak modulation on a multiple frequency dramatically modifies the basic structure of resonances and allows management of positions and strengths of resonances.

The propagation of laser radiation along the ξ axis of the directional coupler is described by the Schrödinger equation for the dimensionless field amplitude q :

$$i \frac{\partial q}{\partial \xi} = -\frac{1}{2} \frac{\partial^2 q}{\partial \eta^2} - pR(\eta, \xi)q. \quad (1)$$

Here η and ξ are the normalized transverse and longitudinal coordinates, while p stands for the refractive index modulation depth. The refractive index distribution is given by $R(\eta, \xi) = [1 + Q(\xi)] \exp[-(\eta + w_s/2)^6/w_\eta^6] + [1 - Q(\xi)] \exp[-(\eta - w_s/2)^6/w_\eta^6]$, where w_s is the separation between channels and w_η is the channel width. The out-of-phase longitudinal refractive index modulation in the channels is described by $Q(\xi) = \mu \sin(\Omega\xi) + \alpha \sin(n\Omega\xi)$, where μ is the depth of modulation on main frequency Ω , while α is the depth of modulation on frequency with multiplicity n . We set $w_\eta = 0.3$, $w_s = 3.2$, and $p = 2.78$. For characteristic transverse scale $10 \mu\text{m}$ and $\lambda = 820 \text{ nm}$ this describes two $3 \mu\text{m}$ waveguides with $32 \mu\text{m}$ separation and refractive index depth $\sim 3.1 \times 10^{-4}$.

To study beam dynamics we excited the left channel at $\xi = 0$, using the linear guided mode of the isolated waveguide. When $\mu = \alpha = 0$ the light periodically switches between channels with beating frequency $\Omega_b = 2\pi/T_b$, where $T_b = 100$ for our parameters. When $\mu \neq 0$, $\alpha = 0$ the coupling suppression can be achieved for a strictly defined set of modulation frequencies [21]. This is accompanied by a resonant increase of distance-averaged energy flow $U_m = L^{-1} \int_0^L d\xi \int_{-\infty}^0 |q(\eta, \xi)|^2 d\eta / \int_{-\infty}^0 |q(\eta, 0)|^2 d\eta$ trapped in the input channel (here L is the final distance). The primary resonance in the dependence $U_m(\Omega)$ has the largest frequency Ω_{rp} , while frequencies of secondary resonances $\approx \Omega_{rp}/n$, where $n = 2, 3, 4, \dots$. For moderate μ values $\Omega_{rp} \sim \mu$, while for $\mu \rightarrow 0$ one has $\Omega_{rp} \rightarrow \Omega_b/2$.

The central result of this Letter is that the addition of even weak modulation on multiple frequencies drastically modifies the basic resonance structure. Thus, Fig. 1(a) shows dependencies $U_m(\Omega)$ around

the primary resonance for increasing α values and $n=2$. The additional modulation on a double frequency gradually destroys the primary resonance, and resonant value U_{mr} of averaged energy that was ≈ 1 at $\alpha=0$ drops down to $1/2$ already for α values much smaller than μ [Fig. 1(b)]. The frequency Ω_{rp} remains almost unchanged with α . The localization decreases not only in the primary resonance, but also in secondary resonances, although larger amplitudes α are necessary for their destruction. The critical value α_{cr} at which U_{mr} decreases to $1/2$ monotonically grows with μ , but in all cases $\alpha_{cr} \ll \mu$.

The picture is completely different when the main frequency modulation is combined with modulation on triple frequency. This results in a visible increase of strengths of resonances and also leads to a progressive shift in the frequencies of primary and secondary resonances that at $n=3$ occurs in such way that primary resonance frequency Ω_{rp} decreases while secondary resonance frequency Ω_{rs} increases, resulting in a gradual fusion of these resonances [Fig. 1(c)]. The width of such combined resonances is much larger than widths of isolated resonances at $\alpha=0$; i.e., almost perfect coupling suppression occurs for a broader band of modulation frequencies. The broadening of resonances may be potentially employed to achieve coupling suppression for polychromatic radiation. Dependencies of Ω_{rp} , Ω_{rs} , and the difference $\delta\Omega = \Omega_{rp} - \Omega_{rs}$ on amplitude α are shown in Fig. 1(d). The critical amplitude α_{cr} at which $\delta\Omega$ decreases to $\Omega_{max}/2$, where $\Omega_{max} = \delta\Omega_{\alpha=0}$, is a monotonically increasing function of μ . Higher-order resonances also may fuse, although in that case resonant frequencies may depend on α in a complex way.

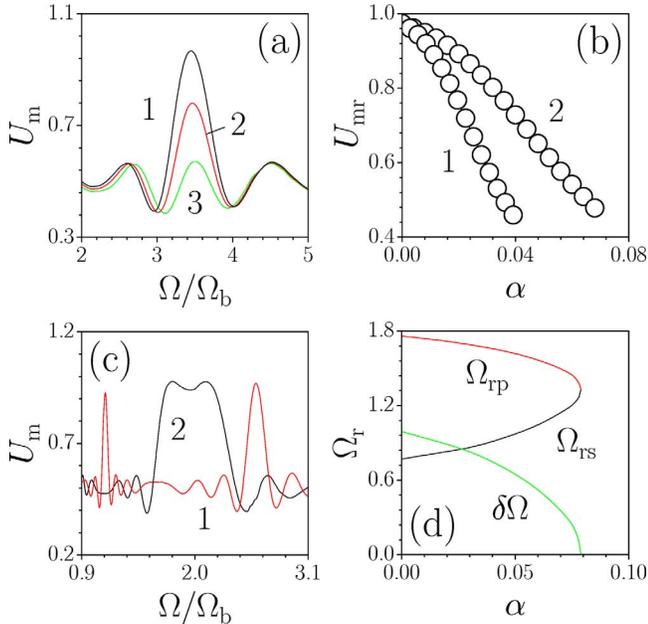


Fig. 1. (Color online) (a) U_m versus Ω at $\alpha=0$ (curve 1), $\alpha=0.025$ (curve 2), and $\alpha=0.040$ (curve 3) for $\mu=0.2$, $n=2$. (b) U_m versus α at $\Omega=\Omega_{rp}$ for $\mu=0.15$ (curve 1) and $\mu=0.30$ (curve 2) at $n=2$. (c) U_m versus Ω for $\alpha=0$ (curve 1) and $\alpha=0.11$ (curve 2) for $\mu=0.15$, $n=3$. (d) Ω_{rp} , Ω_{rs} , and $\delta\Omega = \Omega_{rp} - \Omega_{rs}$ versus α for $\mu=0.1$, $n=3$.

The phase mismatch φ between modulations on different frequencies that may be introduced into the function $Q(\xi) = \mu \sin(\Omega\xi) + \alpha \sin(n\Omega\xi + \varphi)$ notably modifies coupling dynamics. Thus, for $n=2$, $\mu=0.20$, $\alpha=0.03$ the averaged energy in principal resonance grows from $U_m(\varphi=0)=0.69$ to $U_m(\varphi=\pi/2)=0.98$ and then decreases to $U_m(\varphi=\pi)=0.74$. A similar trend was found for $n=3$. This indicates the importance of the concrete functional shape of longitudinal modulation $Q(\xi)$, which strongly depends on phase shift.

Nontrivial features of reshaping of resonance curves may be interpreted in the frames of the tight-binding approximation operating with field amplitudes in adjacent channels $q_1(\xi)$, $q_2(\xi)$, whose dynamics is described by the coupled ordinary differential equations [21]. Introducing symmetric and antisymmetric modes $C_s = (q_1 + q_2)/2$, $C_a = (q_1 - q_2)/2$, one gets

$$\begin{aligned} \frac{dC_s}{d\xi} &= +i\kappa \operatorname{Re}[F(\xi)]C_s - \kappa \operatorname{Im}[F(\xi)]C_a, \\ \frac{dC_a}{d\xi} &= -i\kappa \operatorname{Re}[F(\xi)]C_a + \kappa \operatorname{Im}[F(\xi)]C_s. \end{aligned} \quad (2)$$

Here $\kappa = p \int_{-\infty}^{\infty} w_1 w_2 R_{\xi=0} d\eta$ is the coupling constant, $w_{1,2}$ describe profiles of guided modes in two waveguides and are normalized in such way that $\int_{-\infty}^{\infty} w_{1,2}^2 d\eta = 1$, the old notations μ and α were kept for amplitudes of longitudinal modulation in the tight-binding approximation (they are in fact given by $\mu p \int_{-\infty}^{\infty} w_1^2 R_{\xi=0} d\eta$ and $\alpha p \int_{-\infty}^{\infty} w_1^2 R_{\xi=0} d\eta$), while the real and imaginary parts of the factor $F = \exp[(2i\mu/\Omega)\cos(\Omega\xi) + (2i\alpha/n\Omega)\cos(n\Omega\xi)]$ determine phase shifts of symmetric and antisymmetric modes and their coupling. Single-channel excitation corresponds to $C_s(0) = C_a(0) = 1$, and switching occurs at

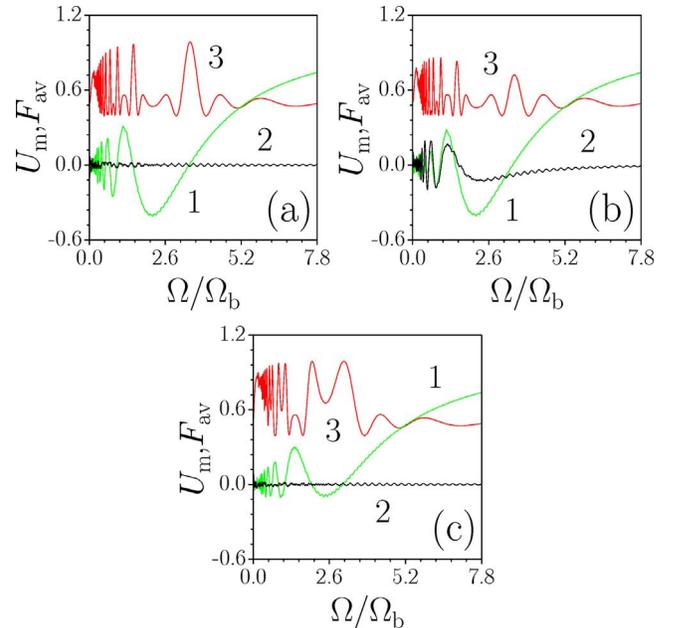


Fig. 2. (Color online) $\operatorname{Re}(F_{av})$ (curves 1), $\operatorname{Im}(F_{av})$ (curves 2) and U_m (curves 3) versus Ω for (a) $\alpha=0.00$, $n=0$, (b) $\alpha=0.40$, $n=2$, and (c) $\alpha=0.15$, $n=3$, at $\mu=0.258$.

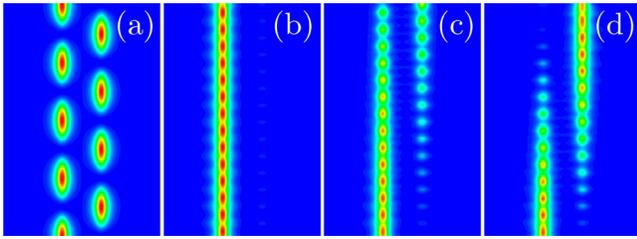


Fig. 3. (Color online) Propagation dynamics in (a) unmodulated coupler at $\mu=0$, $\alpha=0$, and in modulated couplers at (b) $\mu=0.2$, $\alpha=0$, (c) $\mu=0.2$, $\alpha=0.025$, and (d) $\mu=0.2$, $\alpha=0.05$. Here $n=2$, $L=4T_b$. Modulation frequency $\Omega=3.45\Omega_b$ in (b)–(d) corresponds to Ω_{TP} for $\mu=0.2$, $\alpha=0$.

the distance L , where the accumulated phase difference is $\arg[C_s(L)] - \arg[C_a(L)] = \pi$. Importantly, in the presence of refractive index modulation, mode dephasing and mode coupling is suppressed if $\text{Re}(F) \rightarrow 0$ and $\text{Im}(F) \rightarrow 0$. Figure 2 shows real (curves 1) and imaginary (curves 2) parts of distance-averaged factor $F_{\text{av}} = L^{-1} \int_0^L F(\xi) d\xi$ versus Ω in comparison with $U_m(\Omega)$ dependence (curves 3). At $\alpha=0$ the value $\text{Im}(F_{\text{av}})$ is negligible and positions of resonances are defined by zeros of $\text{Re}(F_{\text{av}})$ [Fig. 2(a)]. The modulation on double frequency results in an increase of $|\text{Im}(F_{\text{av}})|$ within the band of primary resonance and gradually destroys this resonance [Fig. 2(b)]. Additional modulation on triple frequency shifts the $\text{Re}(F_{\text{av}})$ curve upward, which diminishes the interval between zeros of $\text{Re}(F_{\text{av}})$ and causes fusion of primary and secondary resonances [Fig. 2(c)].

Typical propagation scenarios in couplers with biharmonic modulation are presented in Figs. 3 and 4. While in the presence of longitudinal modulation with only frequency the periodic energy exchange [Fig. 3(a)] between channels of coupler may be almost completely suppressed in resonance [Fig. 3(b)], the addition of modulation with $n=2$ destroys localization as long as α grows [Figs. 3(c) and 3(d)]. In contrast, addition of modulation with $n=3$ leading to shift and broadening of resonances may result in suppression of coupling [Figs. 4(b) and 4(d)] at modulation frequencies that are well outside any resonances in system with $\mu \neq 0$, $\alpha=0$ [Figs. 4(a) and 4(c)]. The combination of main modulation with modulation at much higher frequencies with $n=4, 5, \dots$ is not effective, because such high-frequency oscillations aver-

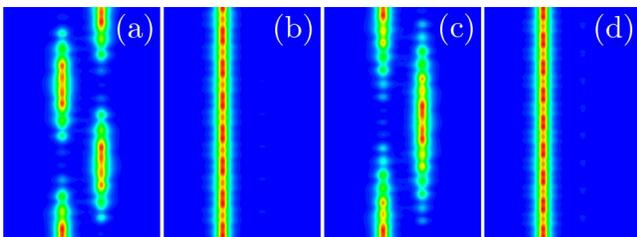


Fig. 4. (Color online) Propagation dynamics in modulated couplers with (a) $\alpha=0.00$, $\Omega=1.78\Omega_b$, (b) $\alpha=0.11$, $\Omega=1.78\Omega_b$, (c) $\alpha=0.00$, $\Omega=2.11\Omega_b$, and (d) $\alpha=0.11$, $\Omega=2.11\Omega_b$. Here $\mu=0.15$, $n=3$, $L=4T_b$.

age out, and one has to increase α considerably in order to affect resonance structure—something that also leads to growth of radiative losses.

Summarizing, we found that biharmonic longitudinal refractive index modulation in couplers allows management of positions and strengths of resonances where coupling suppression is achieved.

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