Twisted photons: new classical and quantum applications

Juan P. Torres, Gabriel Molina-Terriza and Lluis Torner
ICFO-Institut de Ciencies Fotoniques, Barcelona, Spain
Dep. Signal Theory and Communications, Universitat Politècnica de Catalunya, Barcelona, Spain

ABSTRACT

Twisted light, or light with orbital angular momentum (OAM), plays an emerging role in both classical and quantum science, with important applications in areas as diverse as biophotonics, micromachines, spintronics, or quantum information. It offers fascinating opportunities for exploring new fundamental ideas in physics, as well as for being used as a tool for practical applications. One important point is to determine how to generate single photons, and two-photon states, with an appropriate OAM content. Here we describe the paraxial orbital angular momentum of entangled photon pairs generated by spontaneous parametric down-conversion (SPDC) in different non-collinear geometries. These geometries introduce a variety of new features. In particular, we find the OAM of entangled pairs generated in purely transverse-emitting configurations, where the entangled photons counter-propagate perpendicularly to the direction of propagation of the pump beam. The spatial walk-off of all interacting waves in the parametric process also determines the OAM content of the down-converted photons, and here its influence is also revealed.

Keywords: quantum optics, nonlinear optics, parametric processes, entanglement, orbital angular momentum.

1. INTRODUCTION

Spontaneous parametric down conversion (SPDC) is a reliable source of entangled photons. The generated two-photon state can be entangled, e.g., in polarization, or spin angular momentum (SAM), or in frequency. It can be also entangled in paraxial orbital angular momentum (OAM). The corresponding multidimensional entangled states, or qudits, provide higher dimensional alphabets, thus enhancing the potential of quantum techniques.

The angular momentum of light contains a spin contribution, dictated by the polarization of the electromagnetic fields, and an orbital contribution, related to their spatial structure. In general, only the total angular momentum is an observable quantity. However, within the paraxial regime, both contributions can be measured and manipulated separately. While the spin angular momentum is extensively employed in quantum information schemes, only recently the orbital angular momentum (OAM) has been added to the toolkit.

To date, most investigations addressed SPDC in nearly collinear phase-matching geometries, where the pump, the signal and idler photons propagate almost along the same direction. However, non-collinear geometries introduce a variety of new features. In the quantum domain, it has been shown that when the counter-propagating entangled photons are emitted in a thin waveguide or a waveguide with periodic nonlinearity, their spectral bandwidth drastically decreases. The OAM of the entangled photons strongly depend on the propagation direction of the photons and the geometry considered. Here we show that locally paraxial measurements of the OAM conducted with entangled photons generated in noncollinear geometries, they do not comply with the known selection rules for the spiral index of the pump, signal and idler mode functions, as stated in Mair et al., Nature 412, 313 (2001).

We will consider geometries where both downconverted photons counterpropagate, and we will derive a new selection rule for OAM for the case where all interacting beams propagate coaxially with the pump beam. In particular, in transverse emitting configurations, the spatial shape of the down converted in one transverse dimensions strongly depends on the corresponding spatial shape of the input pump beam, while in the other transverse dimension, the shape is tailored by the longitudinal phase matching.
2. GENERAL FORMALISM

2.1 OAM for a single photon

The quantum state $|\Psi\rangle$ of a photon is described by a mode function. Within the paraxial regime of light propagation, any mode function with an arbitrary amplitude profile can be expanded into Laguerre Gaussian (LG) modes, so that the quantum state of the photon can be written as

$$|\Psi\rangle = \sum_{l=-\infty}^{\infty} \sum_{p=0}^{\infty} C_{lp} |l, p\rangle$$

where the quantum state $|l, p\rangle$ is given by

$$|l, p\rangle = \int dq LG_{lp}(q) a^+(q) |0\rangle$$

where $|0\rangle$ is the vacuum state and $a^+(q)$ is the creation operator for a photon with transverse wave vector $q$. The normalized LG mode at its beam waist ($z=0$) in the spatial frequency domain is given by

$$LG_{lp}(\rho_k, \varphi_k) = \sqrt{\frac{w_0^2}{2\pi(l+p)!}} \frac{1}{w_0 \sqrt{2}} L_p^l \left( \frac{w_0^2 \rho_k^2}{2} \right) \exp \left( -\frac{w_0^2 \rho_k^2}{4} \right) \exp \left( il\varphi_k + i\left( p - \frac{l}{2} \right)\pi \right)$$

with $\rho_k$ and $\varphi_k$ being the modulus and phase, respectively, of the transverse components of the wave vector. The functions $L_p^l$ are the associated Laguerre polynomials, $w_0$ is the beam width, $p$ is the number of non-axial radial nodes of the mode and the index $l$, referred to as the winding number, describes the helical structure of the wave front around a phase dislocation. When the mode function is a pure LG mode with winding number $l$, the quantum state is an eigenstate of the OAM operator with eigenvalue $il\hbar$.

State vectors which are not represented by a pure LG mode correspond to photons in a superposition state, with the weights of the quantum superposition dictated by the contribution of the $l$-th angular harmonics. Such superpositions can be restricted to a finite number of modes, or it can consist of an infinite, but discrete, number of modes. The OAM content of the quantum state is then given by the array $P_l = |C_l|^2$. The value of $C_l$ is given by

$$C_l = \int_0^{\infty} \rho_k \, d\rho_k \, |a(\rho_k)|^2,$$

where

$$a(\rho_k) = \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} \Psi(\rho_k, \varphi_k) \exp(-il\varphi_k)$$

and $\sum_{l=-\infty}^{\infty} C_l = 1$, if the mode function $\Psi(\rho_k, \varphi_k)$ is appropriately normalized.

Photons that carry angular momentum in a superposition state of an infinite, but controllable, number of normal modes can be prepared in a variety of ways. An experimentally important scheme is obtained by passing a pure state $|l = 0, p = 0\rangle$ through astigmatic optical components or appropriately designed computer generated holograms. A cylindrical lens imparts a parabolic phase to an incoming photon, therefore, a cylindrical lens prepares photons in a OAM superposition state. The generation of general superposition states of OAM can be achieved by using light vortex-pancakes made of properly distributed single-charge screw dislocations nested into a host Gaussian beam. Such approach relies on the manipulation of the information imprinted on the classical beam that pumps the nonlinear crystal, and makes possible to generate arbitrary qudits in arbitrary Hilbert dimensions.
2.2 The OAM of a two-photon state
The quantum state of the generated two-photon pair is given by
\[
|\Psi\rangle = \int dq_s dq_i \Phi(q_s, q_i) a_s^+(q_s) a_i^+(q_i) |0,0\rangle
\]
where $|0\rangle$ is the vacuum state, $q_s, q_i$ are the transverse components of the signal and idler wave vectors, and $a_{s,i}^+$ are the creation operators for a signal and idler photon, respectively. Normalization of the state requires
\[
\int dq_s dq_i |\Phi(q_s, q_i)|^2 = 1.
\]
One can decompose the quantum state in the base of the eigenstates of the OAM operator as
\[
|\Psi\rangle = \sum_{l_1, p_1, l_2, p_2} C_{l_1, p_1, l_2, p_2}^{l_2, p_2} |l_1, p_1 >, l_2, p_2 >
\]
where
\[
C_{l_1, p_1, l_2, p_2}^{l_2, p_2} = \int dq_s dq_i \Phi(q_s, q_i) [LG_{l_1, p_1}(q_s)]^* [LG_{l_2, p_2}(q_i)]^*
\]
If the idler photon is projected into the quantum state $|l, p >$, which is given by
\[
|l, p >= \int dq LG_{l_0, p}(q) a_i^+(q) |0\rangle
\]
The signal photon turns out to be
\[
|\Psi_s >= \int dq_s \Phi_s(q_s) a_s^+(q_s) |0\rangle
\]
where
\[
\Phi_s(q_s) = \int dq_i \Phi(q_s, q_i) [LG_{l_2, p_2}(q_i)]
\]
Now, as it was described above, the OAM content of the quantum state of the signal photon given by the array
\[
P_i = |C_i|^2.
\]
The value of $C_i$ is given by
\[
C_i = \int_0^\infty \rho_s d \rho_i |a(\rho_i)|^2,
\]
where
\[
a(\rho_i) = \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} \Phi_s(\rho_i, \varphi_i) \exp(-i l \varphi_i)
\]
and $\sum_{l=-\infty}^{\infty} C_l = 1$.

2.3 OAM of a two-photon state in a general non collinear SPDC
We consider a nonlinear optical crystal, illuminated by a quasi monochromatic laser pump beam propagating in the $z$ direction, as shown in Figure 1. The output surface of the nonlinear crystal is assumed to be located at $z=0$. The two-photon quantum state $|\Psi\rangle$ at the output of the nonlinear crystal, can be described by an effective Hamiltonian $H_0(t)$ in the interaction picture, given by
\[
H_I = \epsilon_0 \int dV \chi^{(2)} E_p^+ E_s^+ E_i^+ + c.c.
\]
where $E_p^+$ refers to the positive-frequency part of the pump electric field operator and $E_{s,i}^+$ refers to the negative-frequency part of the signal and idler electric field operators. The amplitude field profile of the paraxial pump beam, which is treated classically, writes
\[
E_p^+(x, z) = \int dq_s dq_i E_0(q_s, q_i) \exp[i k_p(q_s, q_i) z + i q_s [x + z \tan \rho_{0x}] + i q_i [y + z \tan \rho_{0y}]] - i \omega_p t
\]
where $\omega_p$ is the angular frequency of the pump beam, $k_p(q) = \left(\omega_p^2 n_p^2 / c^2 - |q|^2\right)^{1/2}$ is the wave number inside the crystal, $q = (q_x, q_y)$ is the transverse spatial frequency, $(x, y)$ is the position in the transverse plane, $n_p$ is the refractive index at the pump wavelength, and $E_0(q)$ is the field profile of the pump beam in the spatial frequency domain. The value of the spatial walk off of the pump beam in the x- and y- directions is given by $\rho_0 = (\rho_{0x}, \rho_{0y})$.

![Figure 1. Sketch of the non-collinear down-conversion geometry, showing the propagation directions of the pump, signal, and idler photons.](image)

In order to derive the spatial structure of the generated two-photon state, we define the transformation

$$x_i = x,$$

$$y_i = y \cos \phi_i + z \sin \phi_i,$$

$$z_i = z \cos \phi_i - y \sin \phi_i,$$

where $\phi_i$ (i=1,2) are the angles formed by the direction of propagation of the pump beam (z) and the direction of propagation of the signal (z_1) and idler photons (z_2), respectively. The electric field amplitude operator corresponding to the signal photon $E_s^-$ can be written as

$$E_s^-(x_1, y_1, z_1) = \int dp a_s^+(p) \exp(-ik_s(p)z- ip_s [x_1 + z_1 \tan \rho_{1x}]) - ip_s [y_1 + z_1 \tan \rho_{1y}] + i\omega_1 t)$$

where $(x_1, y_1)$ is the position in the transverse plane of the signal photon, $p = (p_x, p_y)$ is the transverse wave vector for the signal photon, $k_s(p) = \left(\omega_s^2 n_s^2 / c^2 - |p|^2\right)^{1/2}$ is the longitudinal wave number, $a_s^+(p)$ is the creation operator for a signal photon with transverse wave vector $p$, $n_s$ is the refractive index at the signal wavelength. The value of the spatial walk off of the signal photon in the x- and y- directions is given by $\rho_1 = (\rho_{1x}, \rho_{1y})$. Similarly for the idler photon, with $a_s^+(q)$ being the creation operator for an idler photon with transverse wave vector $q$, $n_i$ is the refractive index at the idler wavelength, and the value of the spatial walk off of the idler photon is given by $\rho_2 = (\rho_{2x}, \rho_{2y})$. The signal and idler photons are assumed to be monochromatic, with $\omega_p = \omega_s + \omega_i$. This is justified by the use of narrowband filters in front of the detectors.
Within the first order perturbation theory, the quantum state of the two-photon writes\textsuperscript{14}

$$| \Psi \rangle = \int dpdq \Phi(p,q) a_s^\dagger(p) a_i^\dagger(q) |0,0\rangle$$

with

$$\Phi(q_1,q_2) = E_0(p_x + q_x, \Delta_0) \sin c \frac{\Delta k L}{2} \exp \left(-i \frac{\Delta k L}{2}\right)$$

where $\Delta k$, which comes from the phase matching condition in the z direction, it writes

$$\Delta k = k_p + (p_x + q_x) \tan \rho_0 - p_x \tan \rho_1 - q_x \tan \rho_2 - k_x \cos \varphi_1 - k_x \cos \varphi_2 - p_x \sin \varphi_1 - q_x \sin \varphi_2 - \frac{2\pi}{\Lambda}$$

and

$$\Delta_0 = p_y \cos \varphi_1 + q_y \cos \varphi_2 - k_x \sin \varphi_1 - k_x \sin \varphi_2$$

We have assumed that $\rho_0 = (\rho_0,0), \rho_1 = (\rho_1,0)$ and $\rho_2 = (\rho_2,0)$. A more general case can also be considered.\textsuperscript{15}

The longitudinal wave number writes

$$k_p = \left[ (\omega_p n_p / c)^2 - (p_x + q_x)^2 - \Delta_0^2 \right]^{1/2}$$

In order to fulfill the phase-matching conditions, one can make use of quasi-phase-matching. The period $\Lambda$ is given by

$$\Lambda = \frac{\lambda_p}{n_p - n_x \cos \varphi_1 - n_x \cos \varphi_2}$$

where $\lambda_p$ is the wavelength in vacuum of the pump beam. For a collinear configuration $\varphi_1 = \varphi_2 = 0$, and making use of the paraxial approximation, one obtains the well known result\textsuperscript{12,16}

$$\Phi(p,q) \propto E_0(p+q) \sin c \left( \frac{|p-q|^2 L}{4k_p} \right) \exp \left(-i \frac{|p-q|^2 L}{4k_p} \right)$$

where $k_p = \omega_p n_p / c$. In the thin crystal approximation, one can write\textsuperscript{17} $\Phi(p,q) \propto E_0(p+q)$. As one can see from the previous equations, the orbital angular momentum content of the down converted photons depend on the general nonlinear configuration chosen: the angle of emission of the down converted photons, the width of the pump beam, the length of the crystal and the value of the spatial walk-off of all interacting waves.

3. THE OAM IN NONCOLLINEAR SPDC

Let us consider a non collinear SPDC process in LiIO3. If the optics axis of the nonlinear crystal forms an angle of $\theta_0 = 90^\circ$ with the direction of propagation of the pump beam (axis z), the down converted photons are emitted in the with an angle $\varphi_1 = -\varphi_2 = 17.1^\circ$. This configuration was considered some years ago to study the coherence properties of entangled two photon states.\textsuperscript{18} For the sake of simplicity, we consider a pump wavelength of $\lambda_p = 405$ nm and signal and idler wavelength $\lambda_s = \lambda_i = 810$ nm. For this configuration, the spatial walk off is zero for all interacting waves. Let us assume that the idler photon is projected into a plane wave mode with $q = 0$, which is equivalent to projecting the idler photons into a very large Gaussian beam (OAM of the idler photon, $l_2 = 0$). This projection can be observed experimentally with the idler photon traversing an appropriate 2-f.
Figure 2. Spatial shape and OAM decomposition of the mode function of the signal photon. (a) Spatial shape of the mode function, (b) OAM decomposition. Parameters: Length of the crystal $L=5$ mm, angle of emission of the down converted photons $\phi_1=\phi_2=17.1^\circ$. The pump beam is a Gaussian mode ($l_0=0$) with beam width $w_0=100$ $\mu$m. Focal length of the 2-f system: $f=25$ cm.

Figure 2 shows the spatial shape and mode decomposition of the mode function of the signal photon when the idler photon is projected into $q=0$, the pump beam is a Gaussian mode with $l_0=0$ and the crystal length is $L=5$ mm. Notice that although the signal photon mode decomposition shows a peak at $l_0=0$, many other modes are present in the decomposition, as it is observed in Figure 2(b). This mode decomposition reflects the fact that the mode function shows an elliptical shape, instead of a shape with cylindrical symmetry. This can be easily observed in Figure 2(a). If we reduce the length of the crystal, then the mode function turns is closer to have cylindrical symmetry, as it can be observed in Figure 3 for a crystal length of $L=500$ $\mu$m. Now the OAM decomposition contains just a few modes, and the peak at $l_0=0$ is much more important. The spatial shape is nearly circular now.

The important parameter that determines the influence of the noncollinear angle is the noncollinear length, which writes

$$L_{nc} = \frac{w_0}{\sin \phi_1}$$

If the noncollinear length is much larger than the length of the nonlinear crystal, the noncollinear configuration can be considered as a nearly collinear configuration. If the noncollinear length is smaller than the length of the crystal, this is not longer the case. Figure 2 corresponds to this situation.
Figure 3. Spatial shape and OAM decomposition of the mode function of the signal photon. (a) Spatial shape of the mode function. (b) OAM decomposition. Parameters: Length of the crystal L=0.5 mm, angle of emission of the down converted photons $\phi_1=-\phi_2=17.1^\circ$. The pump beam is a gaussian mode ($l_0=0$) with beam width $w_0=100\mu$m. Focal length of the 2-f system: $f=25$ cm. The nonlinear crystal is LiIO3.

In Figure 4, we consider a thinner crystal (L=100 $\mu$m), but now the pump beam is a vortex beam with $l_0=2$. As it can be seen from Figure 4(b), the OAM mode decomposition corresponds essentially to a single peak at $l_0=2$. In Figure 5, we plot the same case than in Figure 4, but for a thicker crystal (L=2 mm). Now, the effects of the phase matching bandwidth are clearly observed, as it was the case in Figure 2. The main conclusion from Figures 2, 3, 4 and 5, is that for the noncollinear configuration considered (angle of emission $\phi_1=-\phi_2=17.1^\circ$), the effect of the spatial bandwidth of the phase matching function is not negligible. The main parameter that governs its influence is the relationship between the noncollinear length and the nonlinear crystal length. Recently, this effect has been observed experimentally for a $l_0=4$ pump beam in a L=2 mm BBO crystal in a type I configuration\textsuperscript{10}.

We should notice that the observation of this effect depends on the use of a noncollinear configuration, since as it was shown\textsuperscript{12}, in a nearly collinear configuration, when spatial walk off is not present for any of the interacting beam, but phase matching is fully considered, the selection rule

$$l_0 = l_1 + l_2$$

where $l_0$ correspond to the OAM of the pump beam, $l_1$ correspond to the OAM of the signal photon and $l_2$ to the OAM of the idler photon, it is fulfilled. In the experiment made by Mair, Vaziri, Weihs and Zeilinger, all the conditions concerning the crystal length, width of the pump beam and angle of emission of the down converted photons make the OAM of the SPDC to fulfill the above mentioned selection rule. Indeed, in the thin limit crystal approximation is still possible to observe that the OAM of the down converted photons violate the selection rule. For this to be observed, it is required a very large angle of emission of the down converted photons.\textsuperscript{20}
Figure 4. Spatial shape and OAM decomposition of the mode function of the signal photon. (a) Spatial shape of the mode function, (b) OAM decomposition. Parameters: Length of the crystal $L=0.1$ mm. The pump beam has $l_p=2$ with beam width $w_0=100$ $\mu$m.

Figure 5. Spatial shape and OAM decomposition of the mode function of the signal photon. (a) Spatial shape of the mode function, (b) OAM decomposition. Parameters: Length of the crystal $L=2$ mm. The pump beam has $l_p=2$ with beam width $w_0=100$ $\mu$m.
Figure 6. Sketch of the experimental configuration. (a) Non collinear configuration, the relevant parameters are shown as they are inside the crystal, for values outside the crystal the refraction has to be taken into account. (b) Experimental setup. The light from a 405 nm diode laser with 0.6 nm bandwidth is passed through a spatial filter and focused into the LiIO$_3$ crystal. The down converted photons are coupled into multi-mode fibers after traversing through 2-f systems.

Figure 7. Experimentally measured coincidence rate with a pump-beam width (a) $w_p = 32 \mu$m, and (b) $w_p = 500 \mu$m. (c) Width (half width, $1/e^2$) of the conditional coincidence rate along the two transverse directions, x and y, as a function of the pump-beam width. Circles: y dimension; crosses: x dimension; triangles: singles along the x dimension. Solid lines are the best fit to the experimental data. All other experimental conditions as described above.
We set up an experiment (see Figure 6), to show the spatial properties of the photons in a non collinear configuration.\(^{21}\) We use a type-I degenerate non collinear SPDC in a L=5 mm thick lithium iodate (\(\text{LiIO}_3\)) crystal. The crystal is cut in a configuration such that neither of the interacting waves exhibit a Poynting vector walk-off. This is an important point to consider when choosing the most appropriate experimental configuration. For highly focused pump beams, where non collinear effects are to be clearly observable, modifications of the spatial mode function induced by the Poynting vector walk-off are no longer negligible.\(^{15}\) The internal angle of emission of the down converted photons is \(\varphi_1 = -\varphi_2 = 17.1^\circ\), the largest that can be achieved with this crystal.

The pump source is a multimode continuous-wave diode laser emitting light at a central wavelength of \(\lambda_c = 405\) nm, with a bandwidth of about 0.6 nm. The spatial mode at the output of the diode laser is spatially filtered in order to obtain an approximate Gaussian beam with a beam-waist radius of about 500 \(\mu\)m and power up to 25mW. Lenses of different focal length \(f_p\) are placed before the crystal to control the input pump-beam waist.

As depicted in Figure 6, after the crystal a 2-f system of 250 mm focal length is used for each of the down converted beams. The photons are then coupled into multimode optical fibers and sent to the single-photon detectors. The coupling lenses are mounted on a XY translation stage. The one corresponding to the signal-photon beam has been equipped with stepper motors that allow for scanning in the transverse plane. The coupler in the idler beam was used as a fixed reference. Broadband colored filters in front of the couplers are used to remove scattered radiation at 405nm. Pinnholes with diameters of 100 \(\mu\)m and 150 \(\mu\)m (depending on the experiment) are attached to the couplers in order to increase the resolution.

Figure 7 presents the main results of our experiment. We plot the coincidence rate, which corresponds to scanning the signal coupler position along two different orthogonal directions, while keeping fixed the position of the idler coupler. In the x transverse dimension, the spatial shape can well be described within the thin crystal approximation with a Gaussian shape with a beam width of \(w_x = \lambda_c f / (\pi w_0)\). In Fig. 7(a), the pump-beam width, \(w_0 = 32 \mu\)m, yields a non collinear length of \(L_{nc} = 100 \mu\)m, which is much smaller than the crystal length. Fig. 7(b) shows the coincidence rate for a pump-beam width of \(w_0 = 500 \mu\)m. The ellipticity of the state is clearly reduced.

Fig. 7(c) shows the dependence of the width (half width, \(1/e^2\)) of the coincidence rate as a function of the pump-beam width. It shows that the state ellipticity is clearly present when a strongly focused pump beam is used. Small pump-beam widths correspond to small non collinear lengths when compared to the crystal length. With a monochromatic pump and large pump beams we expect the x and y widths to be equal. Nevertheless, a finite spectrum of the pump beam strongly affects the width in the y direction.

4. THE EFFECT OF SPATIAL WALK OFF IN COLLINEAR SPDC

The effect of the non collinear geometry on the OAM of down converted photons can be avoided by making use of collinear geometries, or nearly collinear geometries. But in most cases, the spatial walk off of some, or all, of the interacting beams is still present. We will show that under appropriate conditions, the role of the walk off can not be neglected.\(^{15}\)

Let us consider a nearly collinear geometry \(\varphi_1 = \varphi_2 = 0^\circ\) in type I BBO. The pump beam shows a spatial walk off \(\rho_\varphi = 3.8^\circ\). There is no spatial walk off for the signal and idler photons. Figure 8 shows the spatial shape and mode decomposition of the mode function of the signal photon for a pump beam width of \(w_0 = 50 \mu\)m and \(w_0 = 500 \mu\)m. For \(w_0 = 50 \mu\)m, the mode decomposition is centered at \(l_1 = 2\), but most of the energy is not contained in this mode. We can see that the spatial shape does not have cylindrical symmetry. This is due to the fact that the walk off length of the pump beam \(L_W\) mm, which writes
is smaller than the length of the nonlinear crystals (L=2 mm). For a pump beam width of w₀=50 μm, the walk off length is \( L_\text{w} \approx 0.8 \text{ mm} \). For \( w_0=500 \mu\text{m} \), one has \( L_\text{w} \approx 8 \text{ mm} \).

In order to avoid the specific effects due to the noncollinear configuration and spatial walk off of the interacting beams, quasi-phase matching geometries offers new opportunities. In addition to this, QPM geometries allows to make use of the larger nonlinear coefficient of a given material. In an appropriately chosen QPM configuration, one can erase the effects of non collinear SPDC and of spatial walk off in the OAM of the down converted photons. Figure 9 shows the spatial shape of the mode function and the OAM content of the mode function of the signal photon for SPDC in PPKTP. All interaction waves propagate in the x direction (crystal axis). The pump beam and the signal photon are polarized along the z optics axis, and the idler photon is polarized along the y optics axis. One can observe that the selection rule \( I_0 = I_1 + I_2 \) is fulfilled in this case.

5. NEW STRUCTURES

We consider the orbital angular momentum of entangled photon pairs generated in different geometric configurations where the entangled photons counter-propagate with respect to each other or to the pump (Figure 10). In particular, we find the orbital angular momentum of entangled pairs generated in purely transverse-emitting configurations. Figure 11 shows the spatial shape and mode decomposition of the mode function of the signal photon for some configurations.
depicted in Figure 10, when the pump beam is a vortex beam with \(l_0=1\). The shape of the spatial mode function can be observed by sending the signal photon through an appropriately designed 2-f optical system. Projection of the idler photon into a Gaussian mode implies \(l_2=0\), where \(l_2\) refers to the OAM of the idler photon. In Figure 11(a), we plot the mode decomposition for \(\phi_1 = \phi_2 = 0^\circ\), which shows a single peak at \(l_1=1\). This case correspond to the signal and idler photons propagating in the same direction than the pump beam (forward). Figure 11(c) show the case \(\phi_1 = \phi_2 = 180^\circ\), which corresponds to backward propagation for the signal and idler photons. The mode decomposition shows a single peak that appears at \(l_1=-1\).

For configurations where \(\phi_1 = \phi_2 = 0^\circ\) or \(180^\circ\), which correspond to Figures 10(a), (b) and (c), the condition\(^4^\)

\[
l_0 = s_1 l_1 + s_2 l_2
\]

is fulfilled, where \(s_{1,2} = \pm 1\) correspond to forward (backward) propagation of the corresponding photon. This generalises the selection rule that applies in the collinear case\(^4^\). For \(\phi_1 = -\phi_2 = 90^\circ\), shown in Figure 11(b), the mode decomposition contains several modes. Indeed, the weight of the modes \(l_1=1\) and \(l_1=-1\) are equal. There is no simple relationship between \(l_0\), \(l_1\) and \(l_2\), as it is the case for configurations where the photons co-propagate or counter-propagate coaxially with the pump beam.
Figure 10. Several non collinear geometric configurations for SPDC. In (a) we show the usual collinear configuration for the sake of comparison. In (d) we show the angles $\varphi_1$ and $\varphi_2$.

Figure 11. Spatial shape and OAM decomposition of the mode function of the signal photon. (a) and (b) $\varphi_1 = \varphi_2 = 0^\circ$, (c) and (d) $\varphi_1 = -\varphi_2 = 90^\circ$, and (e) and (f) $\varphi_1 = \varphi_2 = 180^\circ$. Parameters: $l_0=1$, $w_0=300 \, \mu m$ and $L=1 \, mm$. The idler photon is projected into a Gaussian mode ($l_z=0$) with mode width $w_I=300 \, mm$. Notice that the transverse wave vector $(p_x, p_y)$, or momentum, is normalised to a beam width $w=100 \, \mu m$. 
5. CONCLUSIONS

Here we have elucidated the paraxial orbital angular momentum of entangled photon pairs generated by spontaneous parametric downconversion in different noncollinear geometries. We find, in particular, that the orbital angular momentum selection rule of entangled pairs generated in SPDC, namely $l_0 = l_1 + l_2$, it does not hold in general. Notwithstanding, under appropriate conditions that concern the geometric configuration of the SPDC process, the pump beam width and the crystal length, it can be fulfilled.

Noncollinear geometries offer new opportunities for engineering the OAM of entangled photons for quantum information applications. In particular, it can be showed that the paraxial OAM of entangled photon pairs generated in noncollinear geometries where the entangled photons counterpropagate exhibits new features in comparison with those known for collinear geometries. It is also possible to generalize the selection rule connecting the OAM of the pump and the down-converted photons, shown here to hold for co-propagating geometries with no spatial walk off, to different counterpropagating settings where the pump and the generated entangled photons are still coaxial. In non-coaxial, transverse-emitting geometries where the known rules do not hold, the OAM of the entangled photons is dictated by the geometrical ellipticity of the generated spatial two-photon mode function. Importantly, such ellipticity is mediated by the shape of the pump beam and by the length of the nonlinear crystal, in sharp contrast to collinear phase-matching.

Finally, we stress that the effects described here corresponds to quantities that are accessible in current experiments. However, elucidation of the conserved angular momentum of the down-converted photons in non-collinear geometries requires a step forward in the development of the interaction between light and matter. Generally, the question of angular momentum conservation balance requires the simultaneous consideration of the angular momentum of the electronic spins and orbitals, the crystalline structure of the nonlinear crystal and the electromagnetic field.
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